Math 530

2.3 (1ach, 3, 7a)

3.1 (1b, 3, 5a by induction, 11, 19, 29)

3.2 (1bc, 27)

3.3 (no problems)

$T = (V, E)$ is a tree

Def Simple connected undirected graph with no simple circuits.

Th^m ① T has unique simple path between any two v_i.

② remove edge → disconnect

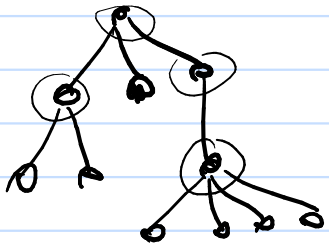
③ add edge → simple circuit

④ $|E| = |V| - 1$

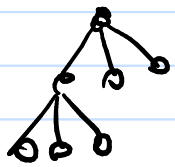
Special trees m-ary | full m-ary trees

m-ary: at most m-children for internal vertices

full m-ary: all internal vertices have exactly m-children



4-ary tree



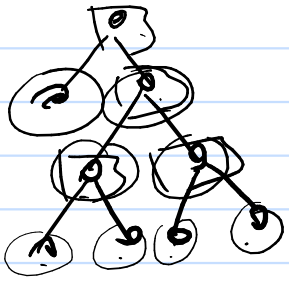
full 3-ary tree

Th^n full m-ary tree

$$|V| = |mi| + 1$$

(root)

all children



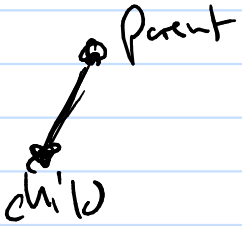
also:

$$|E| = (|V| - 1)$$

sum of number of children

all vert

root



Note: $|V| = n$ $i \equiv$ internal $l \equiv$ leaf

full m-ary

1. $n = i + l$
2. $n = mi + 1$

3 variables n, i, l

together: given one of n, i, l → solve for other 2 in full m-ary tree

Th^n (1) $l \le m^h$ or $h \ge \lceil \log_m l \rceil$

Idea of proof: (by induction) $h = 0$ • so $l = 1$

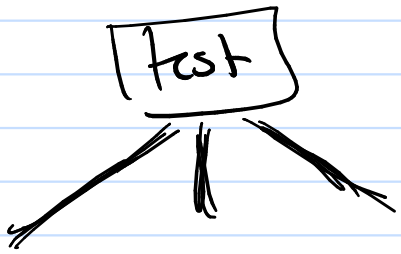
Base:

$l \le m^h$

$1 \le m^0$ true

Inductive (see video)

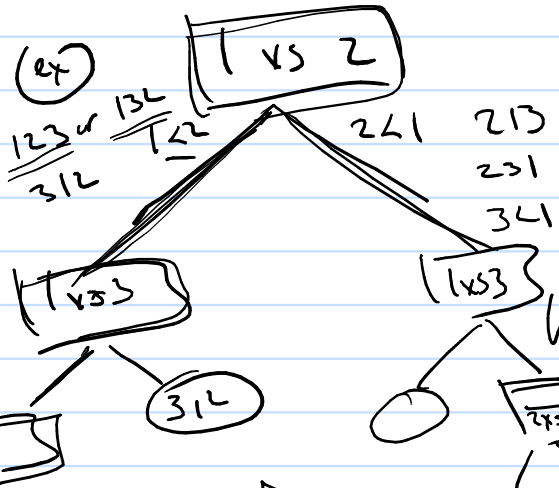
apps ① Decision tree



- internal vertex is a "test" to determine end decision
- leaf ≡ ending outcome

(ex) Sort 3 things a_1, a_2, a_3

outcomes: (123), (132), (213), (231), (312), (321) all leaves



worst case = least height

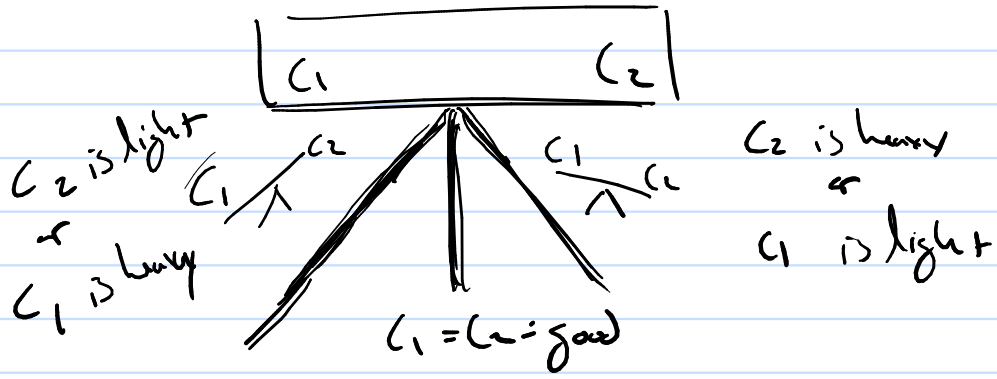
use $h \geq \lceil \log_n l \rceil$

least $h = \lceil \log_n l \rceil$ only happen if full many balanced

(ex) $h = \lceil \log_2 6 \rceil$

$= 3$

other decisions find fake coin by weight

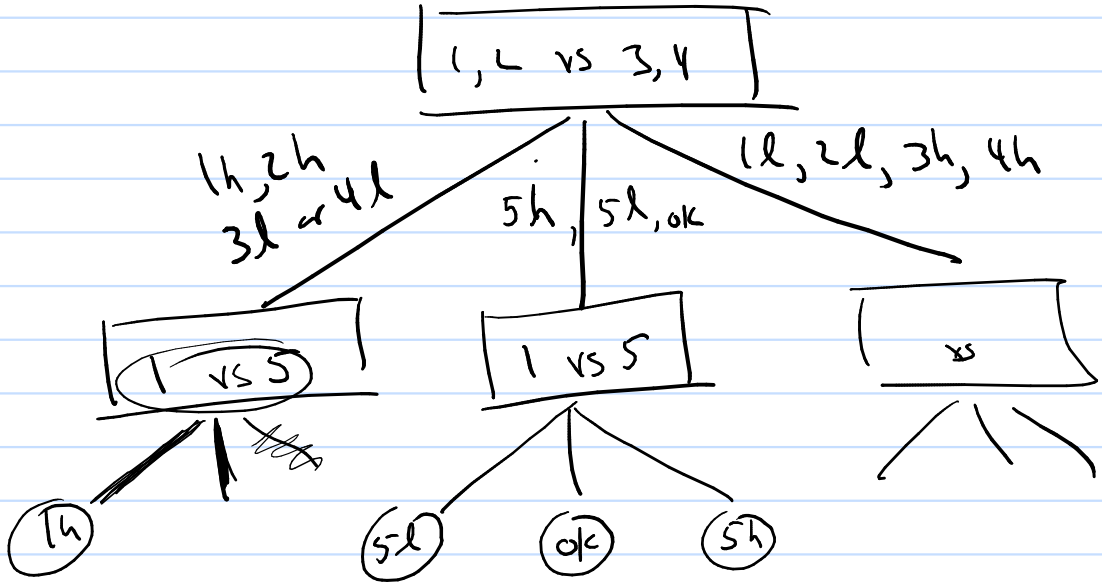


3-ary tree

(ex) 5 coins one may be fake (light? heavy?)
all good?

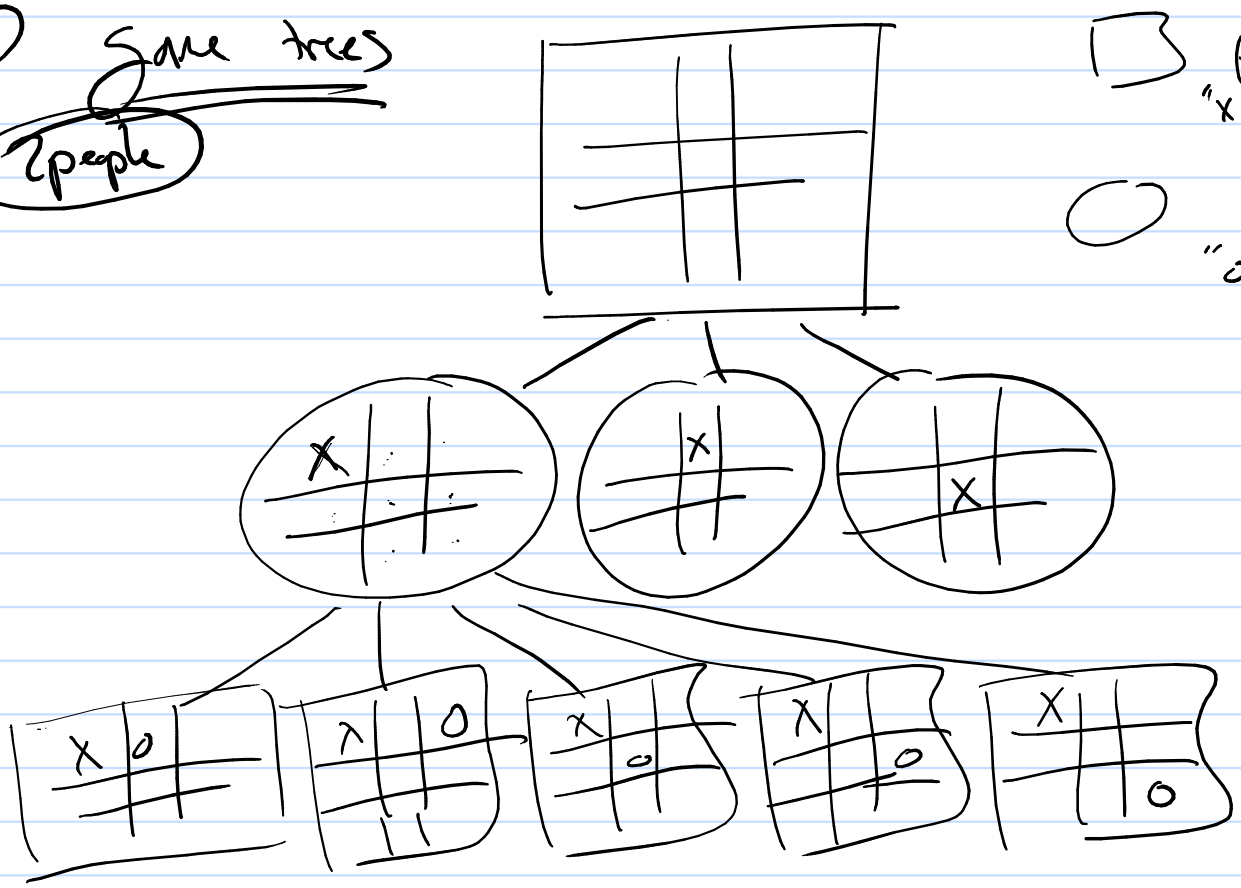
outcomes: $1l, 1h, 2l, 2h, \ldots, 5l, 5h, ok$
$l = 11$

So best case $h = \lceil \log_3 11 \rceil = 3$



(ex) Game trees

2 people



□ player 1 "X" go

○ player 2 "O" go

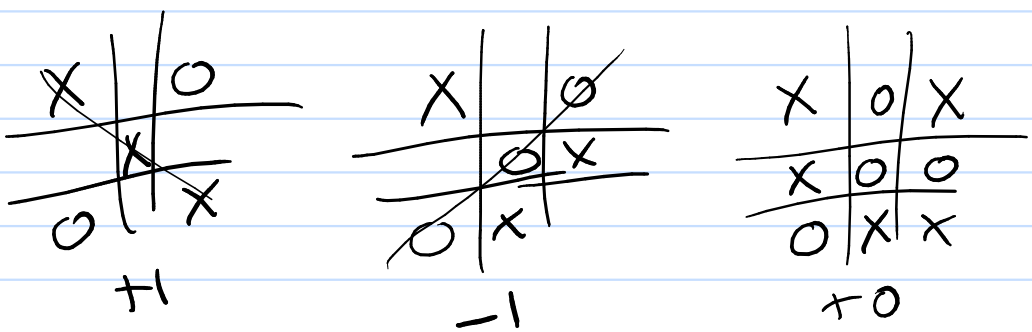
Special feature of game trees is to value [only] vertices (min/max principle)

(1) value of vertex = payout to player 1

(2) each player does what is best for them

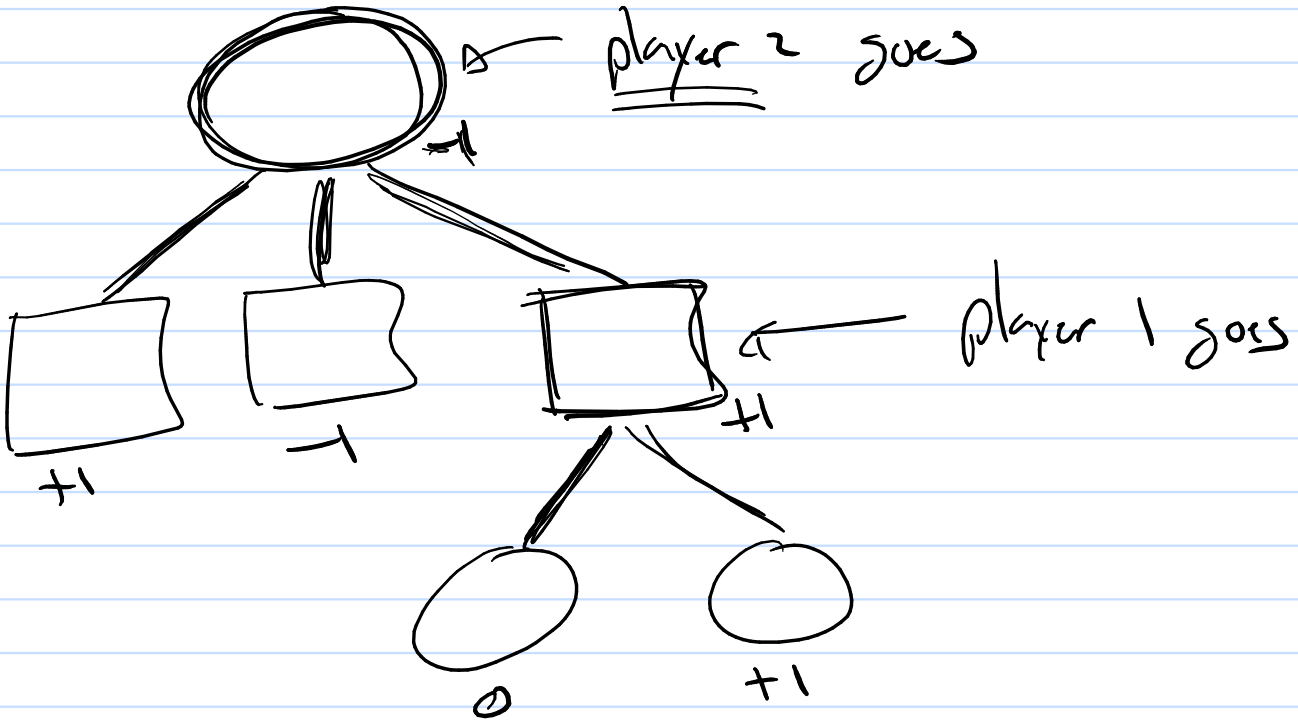
(3) leaf value = payout to player 1

(ex)



+1 −1 +0

(a) Interval vertex value is "players play smart"



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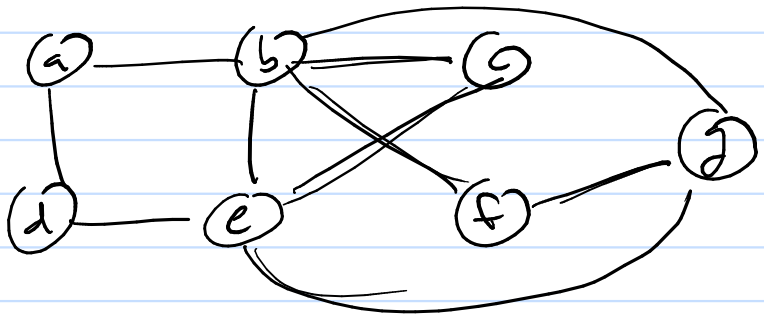
32 Given a connect graph → Make a tree from it

(Spanning trees)

(a)



$|V| = 7$

$|E| = 10$

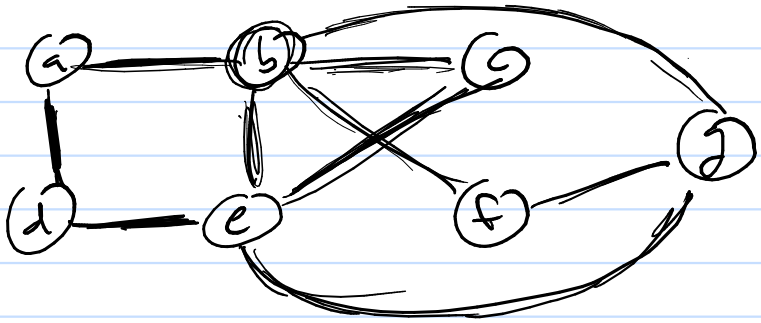
$h \geq \lceil \log_2 7 \rceil$

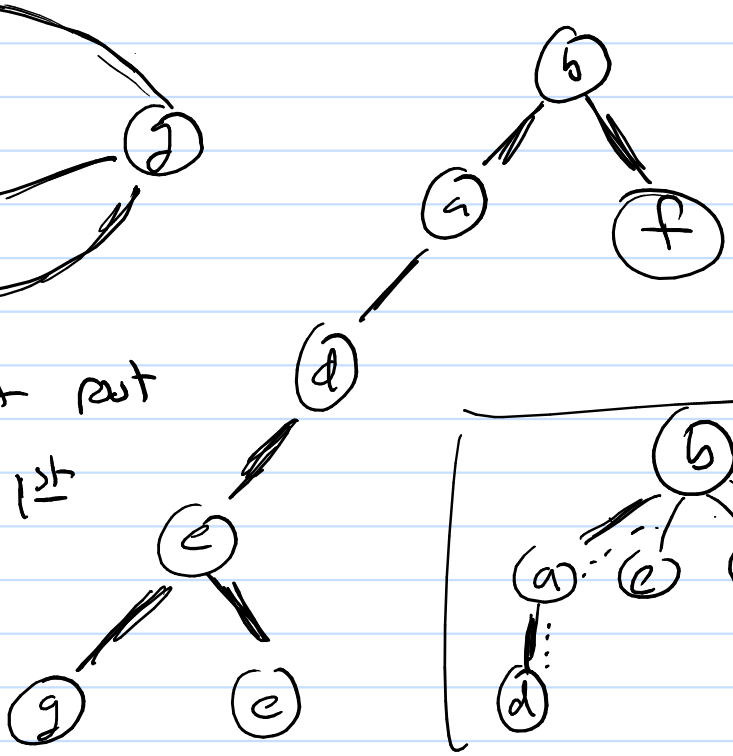
two tech's to find spanning tree of simple graph

(1) depth 1st : focus on found to next series found [m-small, h-large]

(2) breadth 1st : focus on found and all its children

[m-large, h-small]

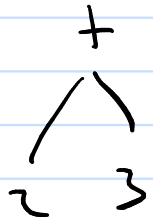




Depth 1st ① get root
② follow depth 1st rules

Breadth 1st ① get root
② follow breadth 1st rules

Next: Notation for 2+3



2, 3, +
+, 2, 3
2, +, 3