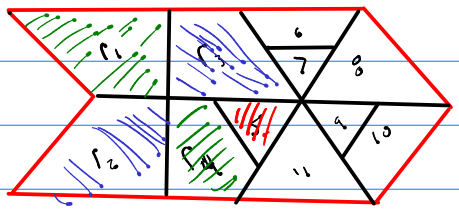
Math 322

Applied Discrete Structures 9.6 Planar Graphs and Coloring Graphs

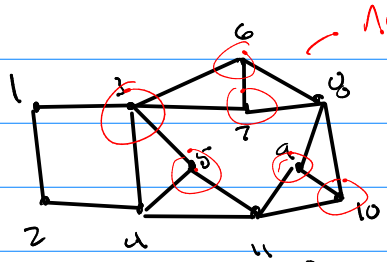
Exam Open Tues (Wed) Next week
- 6 probs
- 4 hours to finish (just to give technical issue time)

Planar Graphs / Graph Coloring

☒ consider a cut puzzle with no outside



Map



no euler path

no euler circuit

simple undirected graph

Before:

Q: Can walk across boundaries = euler path/circuit on graph

9.6 Q1 can G be drawn w/o edges crossing? (planar)

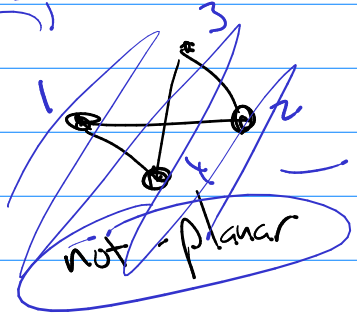
Q2 how many colors for map or vertices so no neighbors have different colors?

Planar Graphs: Simple Undirected graph w/o crossing edges.

ex

planar

Isomorphic



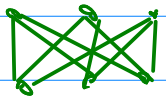
not-planar

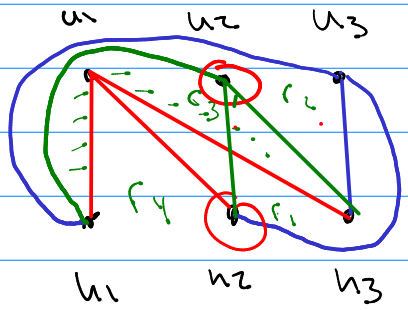
(but isomorphic??)

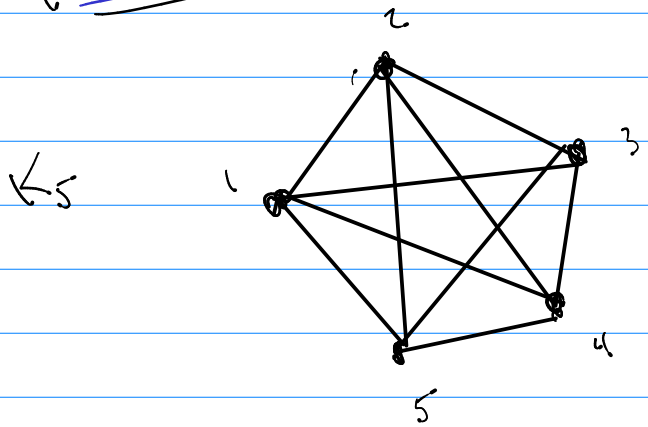
— ignore not-planar isomorphic graphs

example of non-planar $K_{3,3}$

3 utilities puzzle





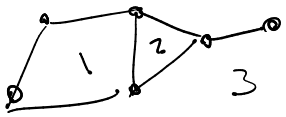


K_5

Kuratowski Reduction thm

$K_{3,3}$ or K_5 is a subgraph in any non-planar graph

Properties of Planar Graphs



$r = 3$
$V = 6$
$E = 7$

(1) Euler's Formula: G is planar $G = (V, E)$

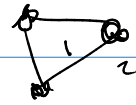
r = regions enclosed by edges

$$\boxed{|V| + r - |E| = 2}$$

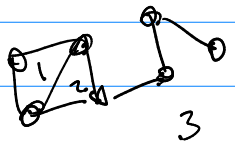
$6 + 3 - 7 = 2$

$3 + 2 - 3 = 2$

$7 + 3 - 8 = 2$



$r = 2$
$V = 3$
$E = 3$



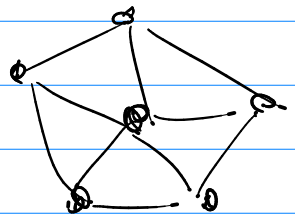
$r = 3$
$V = 7$
$E = 8$

(2) Bounds on edges $|E| \leq 3|V| - 6$

(3) Bound on a vertex's degree

G must have a vertex $\deg(v) \leq 5$

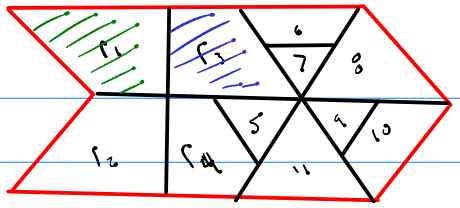
W_5



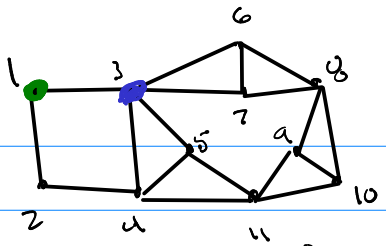
$|V| = 6$ $|E| = 10$

$r = 6$

Coloring:



Map



Simple undirected graph

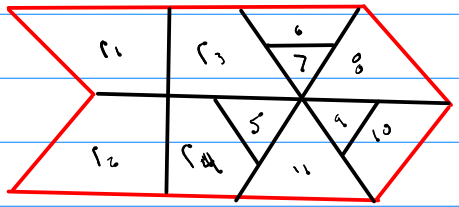
How many colors needed to have different colors for neighbors? Chromatic Number = Smallest amount of colors $\chi(G)$

Facts

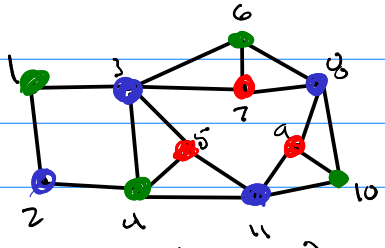
(1) If G is bipartite $\rightarrow$ $\chi(G) = 2$

ex: $K_{3,3}$

(2) 4-Color thm: If G is planar $\chi(G) \leq 4$.



Map



Simple undirected graph

planar so $\chi(G) \leq 4$ | what is $\chi(G) = 3$

play with maps to check..

USA?

Exam Review

6 probs

4 on Boolean Algebra

1 on planar graphs

1 on $\chi(G)$

open tues
close wed 11:30 pm
→ 4 hrs to finish
upload

Boolean Algebra

#1 Verify laws using bit tables

ex) De Morgan's

x	y	$\bar{x}$	$\bar{y}$	$\overline{x+y}$	$\bar{x}\cdot\bar{y}$
1	1	0	0	0	0
1	0	0	1	0	0
0	1	1	0	0	0
0	0	1	1	1	1

Same?!

#2 Verify laws using only

- a) Identity
- b) Complement
- c) Associative
- d) Commutative
- e) Distributive

ex) Idempotent $X = X \wedge X$ $\quad X = X \vee X$

$X = X \wedge 1 = X \wedge (X \vee \bar{X})$

$= (X \wedge X) \vee (X \wedge \bar{X}) = (X \wedge X) \vee 0$

$= X \wedge X$

#3 given $F(x,y,z)$ write it as sum of prods and/or prod of sums

by ...

↑ (1) table

(2) non-table (boolean Algebra laws)

#4 a) Verify NAND and NOR are functionally complete

x	y	F
1	1	0
1	0	1
0	1	1
0	0	0

b) given F write it using only NAND.

#5 Planar graph ... know properties of planar graphs

#6 Find $\chi(G)$ for specific maps / graphs