Math 322

Q's Due Monday
7.1 (1, 3, 7)
7.2 (1, 2, 4, 7, 9)
7.3 (1, 4, 8)

R^* is trans. closure (connectivity relation)

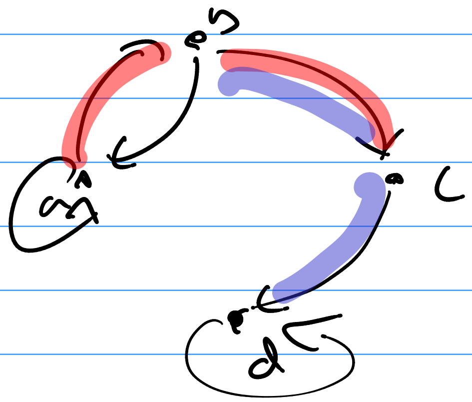
$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee \cdots \vee M_R^{[k]}$$

where $|A| = k$

or

use Warshall's Algorithm.

ex



$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

transitive closure by $M_R^{[k]}$ — paths of length k

a to c

$$M_R^{[2]} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b to d

$$M_R^{[3]} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{M_R^{[2]}} \odot \underbrace{\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{M_R} =$$

Warshals

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad W_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad W_2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_{R^*}$$

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

reflexive closure

$$M_{R\cup\Delta} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

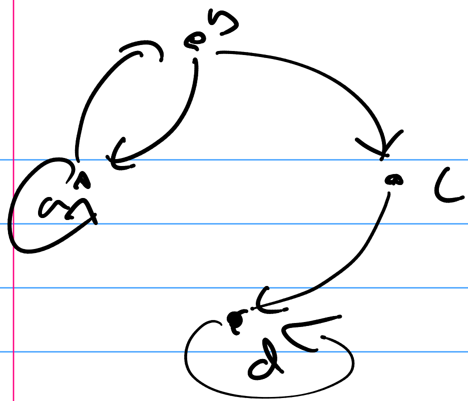
$$\boxed{= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Symmetric Closure

$$M_{R\cup R^{-1}} = M_R \vee M_R^T$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}}$$



$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R = \{ (a,a), (a,b), (b,a), (b,c), (c,d), (d,d) \}$

Function? no b/c $a \to a$, $a \to b$ and $b \to a$, $b \to c$

(it doesn't go to exactly one in codomain)

Exam 1 — 11 probs @ 10pts each

100pts = 100%

Relations

① given R a relation ...

a) state all properties w/ logic

b) logic to english

c) true? false?

ex $R = \{ (a,b) \mid a \text{ has watched person } b \text{ play sports} \}$

reflexive $\forall e \, (eRe)$ everyone has watched themselves play sports.

(2) operators $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \circ R_2$

state as ordered pairs.

(3) list/set builder

Matrix Digraph

given one .. show other two ways to represent R.

(4) be able to find $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \circ R_2$ using matrix ops.

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2}$$

$$M_{R_1 \circ R_2} = M_{R_2} \odot M_{R_1}$$

(5) a) find reflexive closure

b) find symmetric closur

(6) a) trans closure using $M_{R^+} = M_R \vee M_R^{[2]} \vee M_R^{[3]}$

b) trans closure using Warshalls

(7) (8) equiv relation questions

(7) show R $\boxed{\text{IS}}$ an equiv. relation

(8) state equiv. classes

(9) Show R is a partial ordering and draw its Hasse Diagram

(10) You give examples of functions that are...
one-to-one? (yes? no?)
onto? (yes? no?)

(ex) give $f: \mathbb{R} \to \mathbb{Z}$ that is onto and not 1-1

(ex) give $f: \{a, b, c\} \to \{\square, \Delta, ☺\}$ that is not onto and not 1-1

(ex) give a relation from $\mathbb{Z} \to \mathbb{Z}^+$ that is not a function.

$$f(x) = \lceil x \rceil$$

is onto b/c $\lceil \text{int} \rceil = \text{int}$

is not 1-1 b/c $\lceil 1.1 \rceil = \lceil 2 \rceil = 2$

(11) Cardinality $|A| = |B|$ if you can find a bijection from A to B.