Math 322

Note: Exam 1 → 90 pts = 100%

Due Next Wed 9.1 (2, 3, 5, 6, 7, 8)

Exam!

9) $(1, 2, 3, 4, 6, 8, (1))$ ops. = divides $a|b$

ref. $\forall e(eRe) \equiv "e|e"$

→ $e \cdot 1 = e$ so $e|e$ is true

antisym $\forall a \forall b(aRb \wedge bRa \to a=b)$

$\equiv "a|b \wedge b|a \to a=b"$

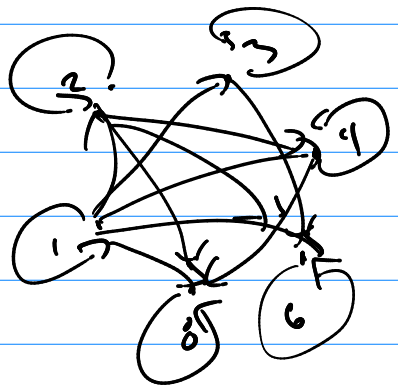
$a \cdot k_1 = b \wedge b \cdot k_2 = a$ so $a \cdot k_1 \cdot k_2 = a$ gives $k_1 = k_2 = 1$ gives $a=b$

trans. $\forall a \forall b \forall c(aRb \wedge bRc \to aRc)$

$\equiv "a|b \wedge b|c \to a|c"$

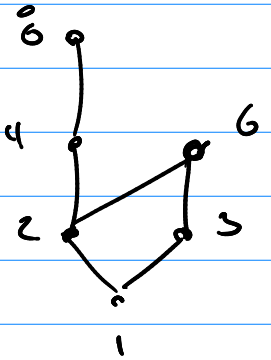
$a \cdot k_1 = b \wedge b \cdot k_2 = c$ so $a \cdot k_1 \cdot k_2 = c \to a|c$

So we have a partial ordering



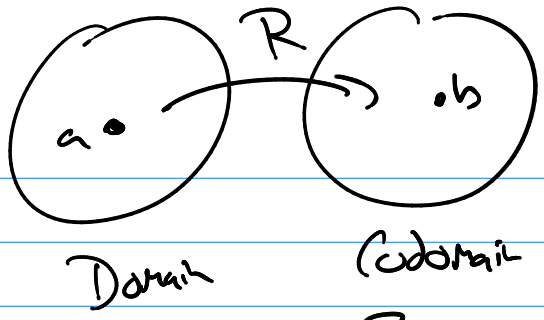
digraph

Hasse



(10) Domain: $\{1, 2, 3, 4, \ldots\}$

Codomain: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$



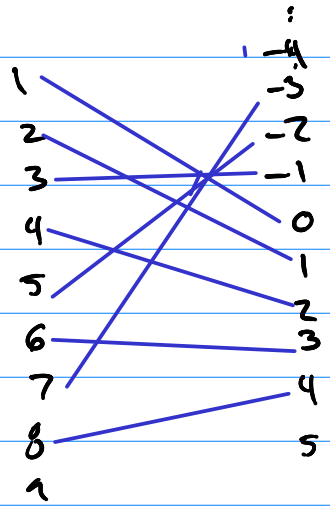
$(a, b) \in R$

a) Not a function

$R = \{(1, 2), (1, -2)\}$

$f(x) = \sqrt{x}$ not a function b/c $\mathbb{Z} \to$ reals!

b) is one-to-one is onto



Verbal

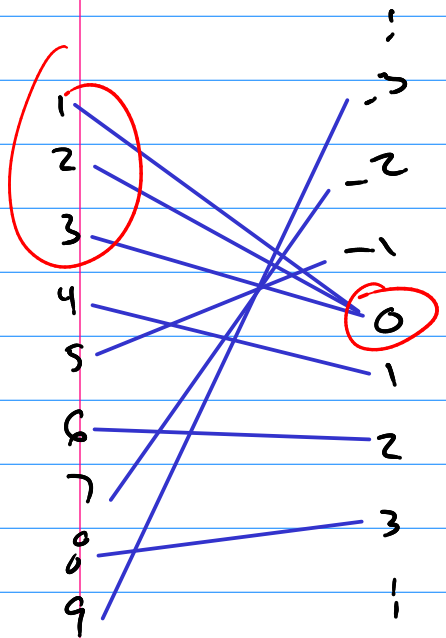
map 1 to 0

all other odds to neg ints

all evens to pos ints

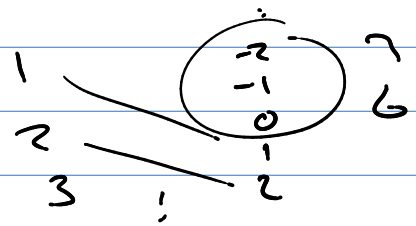
$$f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ -\frac{x-1}{2} & \text{if } x \text{ is odd} \end{cases}$$

c) is onto is not one-to-one



is one-to-one but not onto

$y = x$ not onto



(7) $R = \{ (f,g) \mid f' = g' \}$

ref: $\forall a (aRa) \equiv$ " $f' = f'$ " true.

sym $\forall a \forall b (aRb \to bRa) \equiv$ " $a' = b' \to b' = a'$ " true.

trans $\forall a \forall b \forall c (aRb \wedge bRc \to aRc) \equiv$ " $a' = b' \wedge b' = c' \to a' = c'$ "

true.

(8) $[2x-1] = \{ p \mid p' = z \}$

$P(x) = 2x + c$

---

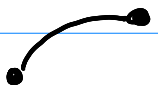
classify graphs $G = (V, E)$

Directed

ordered pairs for E

Undirected

unordered pairs for E



vs

Subtypes

- simple directed
- directed multigraph

has

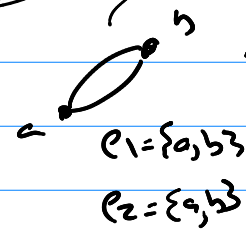
$e_1 = (a,b)$

$e_2 = (a,b)$

$e_3 = (a,b)$

Subtypes

- simple undirected
- undirected multigraph
- pseudograph



also has loops

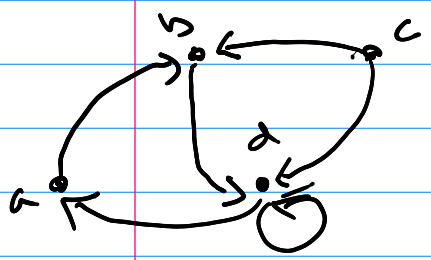
Other features

(1) $|V| \equiv$ number of vertices

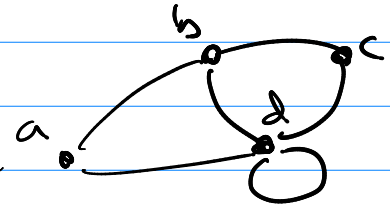
(2) $|E| \equiv$ number of edges

(3) degrees

directed graph vs undirected graph



initial vertex → terminal vertex



out degree $\deg^+(v) =$ # of edges v is initial on

in degree $\deg^-(v) =$ # of edges v is term. on

(ex) $\deg^+(a) = 1$ $\deg^-(a) = 1$

$\deg^+(b) = 1$ $\deg^-(b) = 2$

$\deg^+(c) = 2$ $\deg^-(c) = 0$

$\deg^+(d) = 2$ $\deg^-(d) = 3$

th: $\sum_{v \in V} \deg^+(v) = \sum_{v \in V} \deg^-(v) = |E|$

$\deg(v) =$ number of incident edges (loops count for 2)

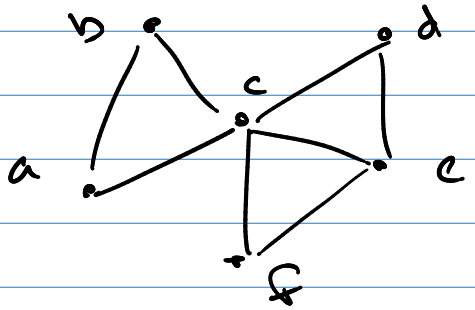
(ex) $\deg(a) = 2$ $\deg(c) = 2$

$\deg(b) = 3$ $\deg(d) = 5$

th: $\sum_{v \in V} \deg(v) = 2|E|$

---

(4) Paths : seq of edges



length = 5

is a path a, b, c, f, c, a

is not a path a, b, c, e ~~a, b~~

$G = (\{a, b, c, d, e, f\}, E)$

is a simple undirected graph