Math 322

Isomorphism

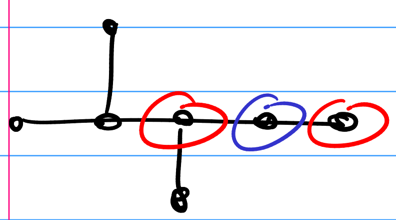
$G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are called isomorphic if

Invariants

1. $|V_1| = |V_2|$, $|E_1| = |E_2|$
2. $\deg(v)$ all match
3. paths match
4. deg & paths and neighborhoods match

and you can find a bijection from V_1 to V_2 that preserves edges.

to show not isomorphic → name broken invariant(s)



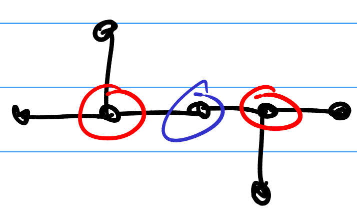
G_1

$|V_1| = 7$

$|E_1| = 6$

degrees?: see 4 of deg = 1
see 1 deg = 2
see 2 of deg = 3

Neighbors:
deg 2 vertex is connected to deg = 3 and deg = 1



G_2

$|V_2| = 7$

$|E_2| = 6$

degrees?: see 4 of deg = 1
see 1 of deg = 2
see 2 of deg = 3

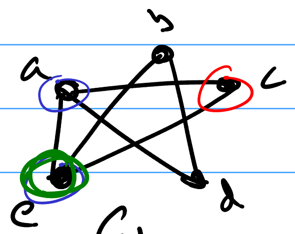
Neighbors
deg 2 vertex is connected to deg = 3 and deg = 3

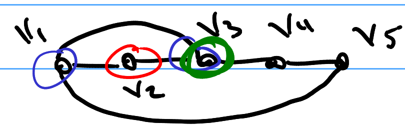
broken invariant so $\boxed{\text{Not}}$ isomorphic.

how to show G_1, G_2 are isomorphic.

Find a bijection from V_1 to V_2 that preserves edges. Call this bijection an isomorphism.

Ex





G_1 $\qquad\qquad$ G_2

$$A$$

$a \longrightarrow v_1$

$b \longrightarrow v_4$

$c \longrightarrow v_2$

$d \longrightarrow v_5$

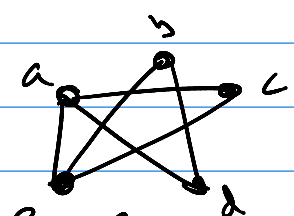
$e \longrightarrow v_3$

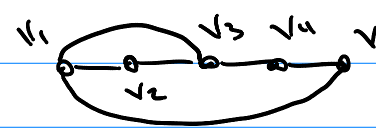
I think this is an isomorphism

$\boxed{\text{check?}}$

need to have a math object that allows obvious equality.

Matrix





G_1 $\qquad\qquad$ G_2

Adjacency Matrix

$$A_{G_1} = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 0 & 1 & 1 & 1 \\ b & 0 & 0 & 0 & 1 & 1 \\ c & 1 & 0 & 0 & 0 & 1 \\ d & 1 & 1 & 0 & 0 & 0 \\ e & 1 & 1 & 1 & 0 & 0 \end{array}$$

$$A_{G_2} = \begin{array}{c|ccccc} & v_1 & v_4 & v_2 & v_5 & v_3 \\ \hline v_1 & 0 & 0 & 1 & 1 & 1 \\ v_4 & 0 & 0 & 0 & 1 & 1 \\ v_2 & 1 & 0 & 0 & 0 & 1 \\ v_5 & 1 & 1 & 0 & 0 & 0 \\ v_3 & 1 & 1 & 1 & 0 & 0 \end{array}$$

Isomorphism order

$$A$$

$a \longrightarrow v_1$

$b \longrightarrow v_4$

$c \longrightarrow v_2$

$d \longrightarrow v_5$

$e \longrightarrow v_3$

$\boxed{\text{Yes}}$ isomorphic

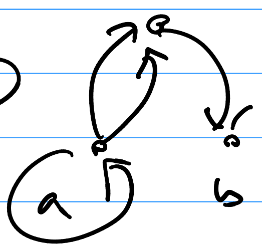
representing graphs in software. $G = (V, E)$

① **Adj. Matrix**

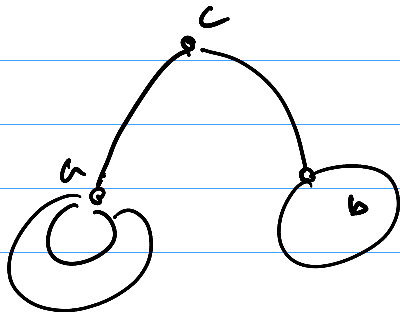
a) order $V = v_1, v_2, \dots, v_n$

b) $A_G = [a_{ij}]$ $\quad a_{ij}$ = # of edges between v_i to v_j

ex



$$A_G = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$$A_G = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Also (paths) $\quad \hat{A}_G = [c_{ij}]$ $\quad c_{ij}$ = # of paths of length n from v_i to v_j

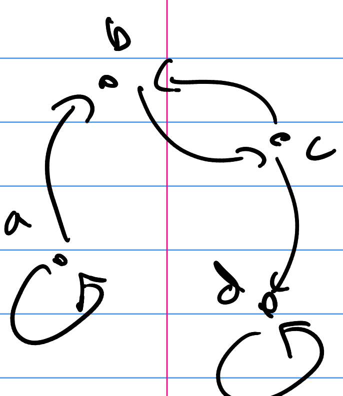
② **edge directory**

a) order $V = v_1, v_2, \dots v_n$

b) edge directory of a single vertex is v_i : [connected vertices]

c) all together

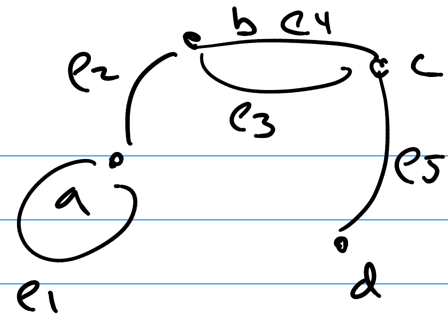
$\{ v_1 : [\,,\,,\,], v_2 : [\,,\,], \dots \}$



$$A_G = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$G = \{ a : [a, b], b : [c], c : [b, d], d : [d] \}$

3) incidence matrices



a) Label V [and] E

b) $I_G = [\alpha_{ij}]$ $\quad \alpha_{ij} = 1$ if v_i and e_j are incident

$\qquad\qquad 0$ otherwise

$$I_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

New

$$\underset{|V| \times |E|}{I_G} \cdot \underset{|E| \times |V|}{I_G^T} = \underset{|V| \times |V|}{m\left[\;\#\;\right]}$$

a variation of the Adj. matrix.

$$\underset{|E| \times |V|}{I_G^T} \cdot \underset{|V| \times |E|}{I_G} = \underset{|E| \times |E|}{\left[\quad\right]}$$

"adjacency in" the idea of adj. edges