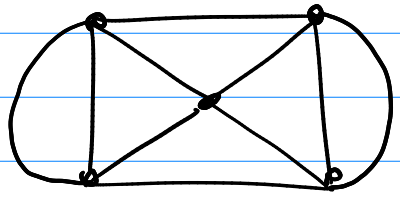
Math 322

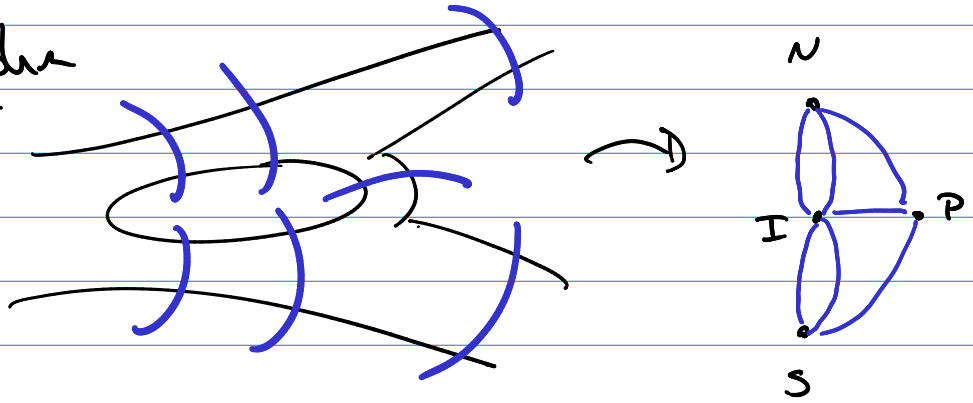
Euler / Hamilton Paths and/or Circuits

Euler Circuit: a path that is...
1. a circuit
2. includes every edge in E exactly once.

(ex) trace problem

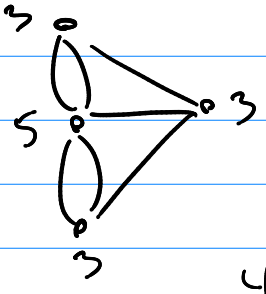


(ex) bridge problem



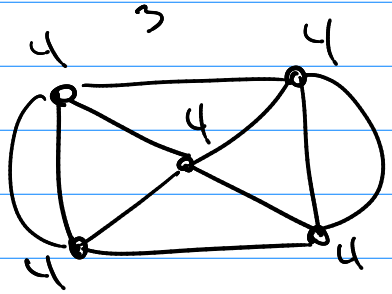
thm $\deg(v)$ for all $v \in V$ is even iff an Euler Circuit exists

So bridge problem:



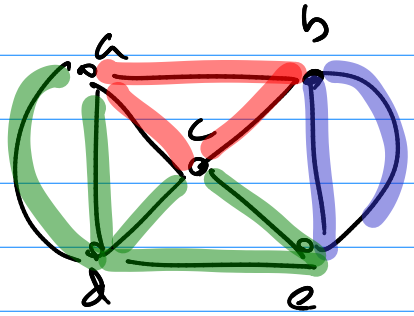
No euler circuit.

trace problem



So has euler circuit.

You find the euler circuit by adding "loops" (circuit) to circuits



a, b, c, a
b, e, b
a, b, e, b, c, a
c, e, d, a, d, c

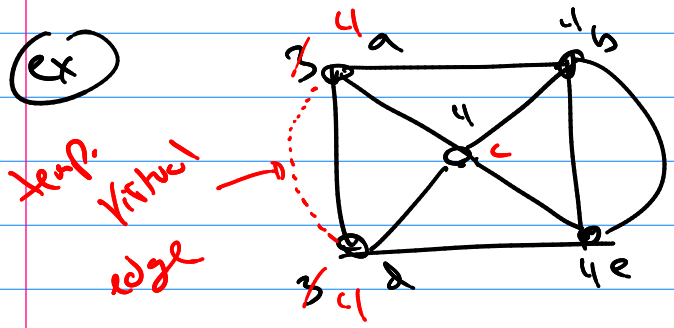
euler circuit a, b, e, b, c, e, d, a, d, c, a

Euler Path

1. path (not circuit)
2. uses all the edges.

th^m all deg(v) are even **except** exactly two are odd **iff** G has an euler path. **and** path starts at one odd ends at the other.

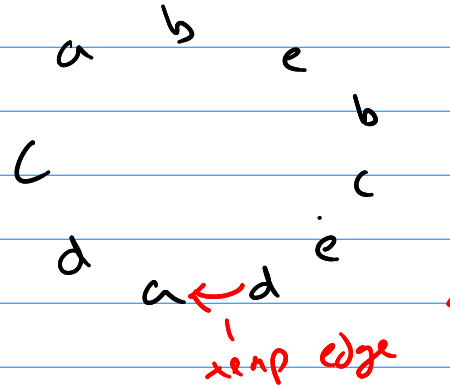
ex



if you add the temp edge → Now we have an euler circuit.

from above --

a, b, e, b, c, e, d, a, d, c, a

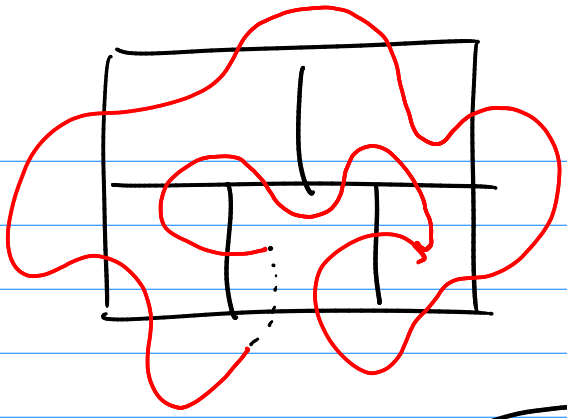


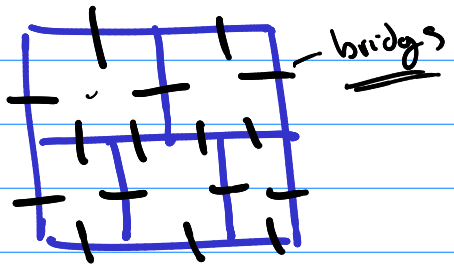
cut the temp edge

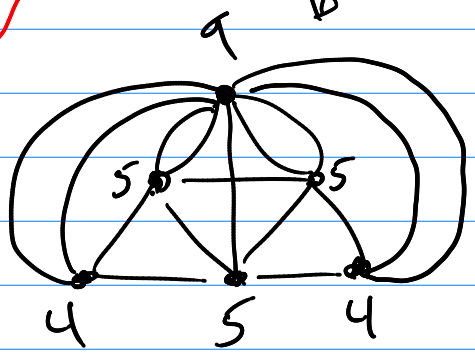
(a), d, c, a, b, e, b, c, e, (d)

given euler path

Cat puzzle





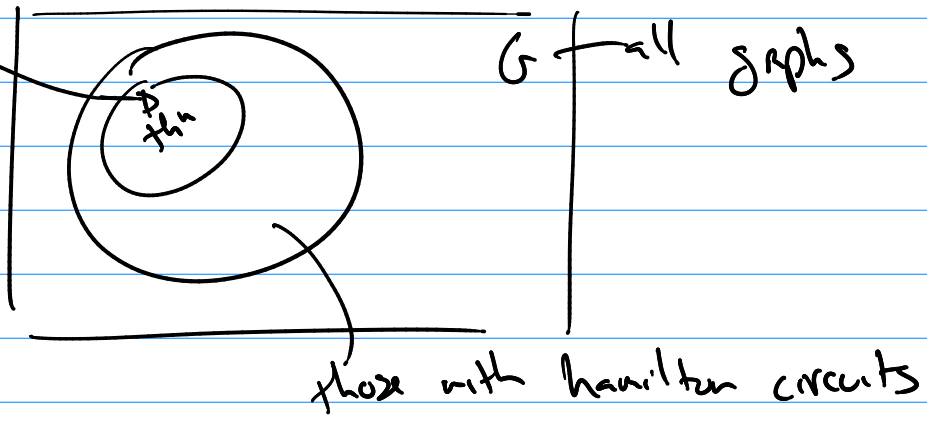


Hamilton circuits : focus on visit all vertices

① visit each vertex exactly once (except the 1st = last)

② path is a simple circuit.

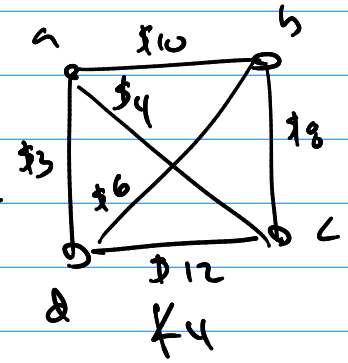
Thm If $\deg(v) \geq |V|/2$ (everyone connects to at least 1/2 of all the vertices)
then Hamilton circuit exists.



- Traveling Salesman problem
- Minimal path problems

Weighted Graph

$G = (V, E)$ with $f : E \to \text{Weights}$



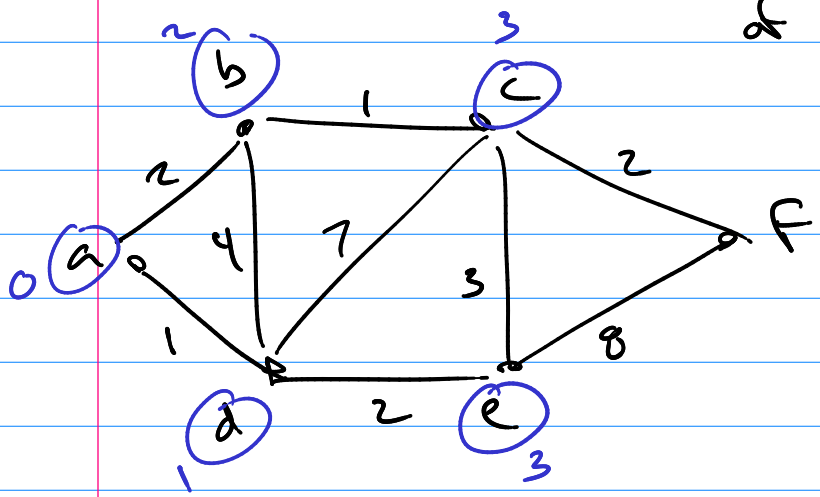
Ex

- length of path = sum of weights along path.
- Note if $f(e) = 1$ this is same as old def. of length.

P length $(a, b, c, d) = \$30$

Dijkstra's Algorithm

(Find all paths from a specific vertex to all the others of least cost)



from a ...

vertex	known path	cost
a	a	0
d	a,d	1
b	a,b	2
c	a,b,c	3
e	a,d,e	3
f	a,b,c,f	5