

# Math 322

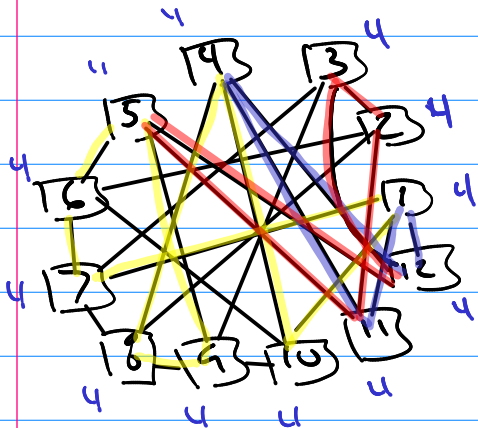
Q's

7.4 #6

Euler

circuit: all even deg

path (7 circuit): all even except 2 odds



deg = blue #

all even so Euler circuit.

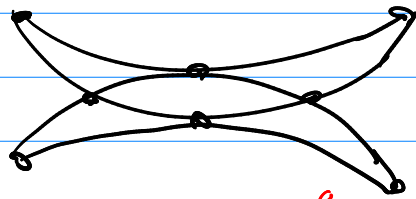
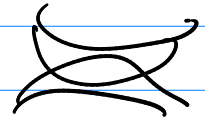
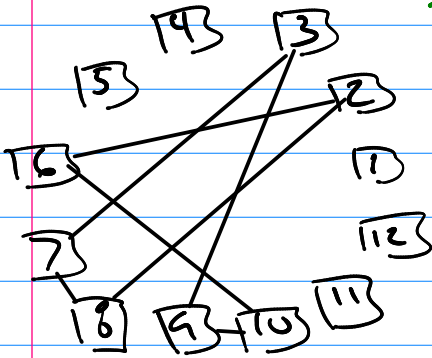
Find an Euler Circuit (connect loops)

1, 7, 6, 5, 9, 8, 4, 10, 1

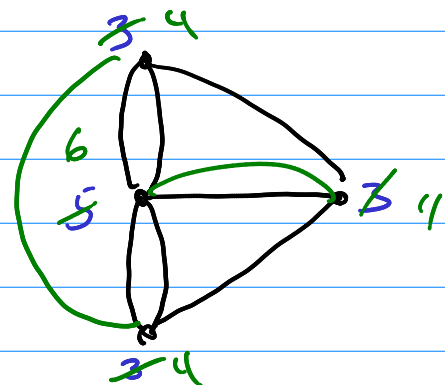
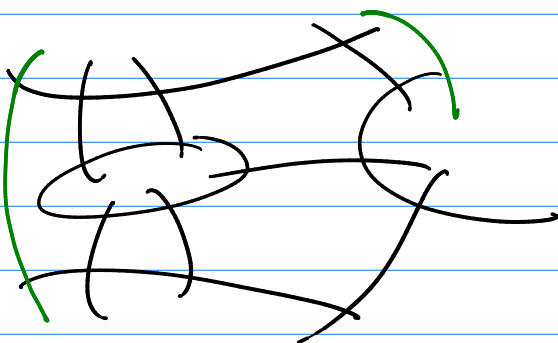
5, 11, 3, 12, 5, 4, 11, 1, 12, 4

2, 6, 10, 9, 3, 7, 8, 2

G' C 6



8	
5	5
4	4



## Dijkstra's Algorithm

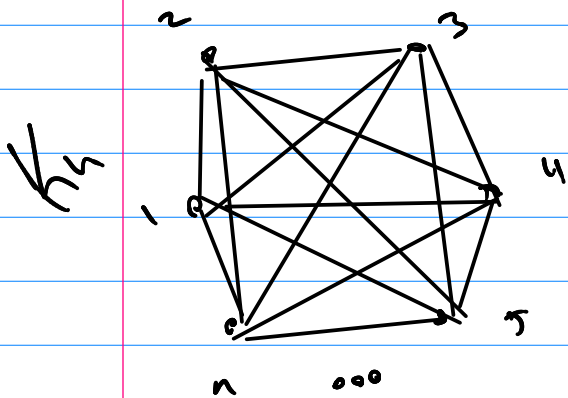
↳ find all minimal paths from a specific vertex to all the other vertices.

↳ find a minimal Hamilton circuit in  $K_n$

(traveling salesman)

thm if  $\deg(v) \geq |V|/2$  for every  $v \in V$

then we have a Hamilton circuit.



$$\deg(v) = n-1 \geq \frac{n}{2}$$

so  $K_n$  has Hamilton circuits

but because it is  $\boxed{K_n}$  we could find all possible Hamilton circuits --

Make our  
Hamilton  
circuit.

Start @  $v_1 \xrightarrow{\text{go to}} ?$  (any one!)  $\rightarrow (n-2) \rightarrow (n-3) \rightarrow \dots$   
 $\uparrow$   $\uparrow$   $\uparrow$   
 $(n-1)$   $n$   $n-1$   
 back to start

$$|\text{Hamilton Circuits}| = (n-1)!$$

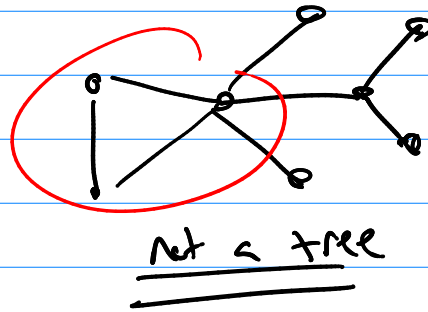
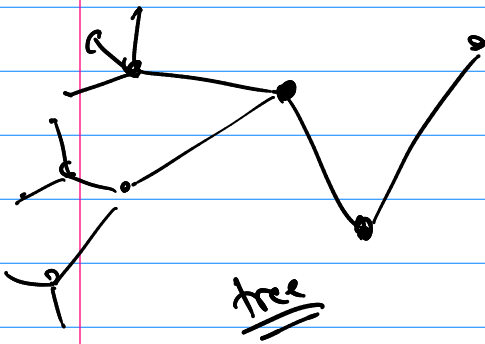
Find the minimal Hamilton circuit  $\rightarrow$

- (1) Make  $\boxed{\text{all}}$   $(n-1)!$  of them
- (2) find all of their lengths
- (3)  $\text{min} = \text{min}$ .

Ch 10

# trees

Def: Undirected graph with no simple circuits is a tree.



Thm the following are logically equivalent

(1)  $G$  is a tree (no simple circuits)

(2) between every distinct pair of vertices there exists a unique simple path.

(3)  $G$  is connected and for every  $e \in E$  if you remove  $e$  to get  $G' = (V, E - \{e\})$  it is disconnected,  $G$  is a tree

(4)  $G$  contains no cycles but adding an edge creates a cycle,  $G$  is a tree

(5)  $|E| = |V| - 1$ ,  $G$  is connected,  $G$  is a tree