Math 322

Discrete 1-2

(Math = toys + rules)

objects — toys; operations, "same" — rules

a) logic
b) number theory
c) set theory
d) relations
e) graph theory
f) → Set to Boolean Algebra

"Algebra"

Math(s) = toys + rules

#1 → non-empty set of elements : V

#2 → operations :

a) unary — takes $e_1 \in V$ gives $e_2 \in V$ (function from V to V)

b) binary — takes $e_1 \in V$, $e_2 \in V$ and gives $e_3 \in V$

(function from $V \times V$ to V)

#3 Possible Properties of operators

Notation: ① use $e_i \in V$
② use $\sim$ for $\sim e_i = e_j$
③ use $\circledast$ for $e_i \circledast e_j = e_k$

① Closure

② Commutative property

$$e_1 \circledast e_2 = e_2 \circledast e_1$$

③ Associative property

$$(e_1 \circledast e_2) \circledast e_3 = e_1 \circledast (e_2 \circledast e_3)$$

(do nothing)

(4) Identity element of V under $\circledast$

(ex) of its π

$$e_i \circledast \pi = e_i$$

(undo)

(5) Inverse element for specific e_i under $\circledast$

$$e_i \circledast (\underset{\text{under } \circledast}{\text{inverse of } e_i}) = \pi$$

(6) Idempotent

$$e_i \circledast e_i = e_i$$

(7) 3 types of distribution — mix 3 elements e_1, e_2, e_3; two binary ops $\circledast_1$ $\circledast_2$

No Assoc? No commutative

left

$$e_1 \circledast_1 (e_2 \circledast_2 e_3)$$

$$= (e_1 \circledast_1 e_2) \circledast_2 (e_1 \circledast_2 e_3)$$

right

$$(e_1 \circledast_1 e_2) \circledast_2 e_3$$

$$= (e_1 \circledast_2 e_3) \circledast_1 (e_2 \circledast_2 e_3)$$

If you have Assoc and commutativity Distribution

$$e_1 \circledast_1 (e_2 \circledast_2 e_3) = (e_2 \circledast_2 e_3) \circledast_1 e_1$$

(8) Involution for a unary operator. ("unary do nothing")

$$\sim(\sim e_i) = e_i$$

to study operators if $|V| = n$ is small

→ we can use tables (ordered lists) to study them.

and	T	F
T	T	F
F	F	F

$(T \wedge T) \to T$

$(T \wedge F) \to F$

$(F \wedge T) \to F$

$(F \wedge F) \to F$