# Math 322

## Relations / Functions

Applied Discrete (Doerr, Levasseur)
ch 6,7

① **N-ary relation**

**Review: Set theory**

(ex) $A = \{1,2,3\}$ $B = \{q, r\}$

$$A \times B = \{(1,q), (1,r), (2,q), (2,r), (3,q), (3,r)\}$$

all ordered pairs $A \times B = \{ (a,b) \mid a \in A \wedge b \in B \}$

all ordered n-tuples $A_1 \times A_2 \times \cdots \times A_n = \{ (a_1, a_2, \dots, a_n) \mid \forall i \; a_i \in A_i \}$

a subset of $A_1 \times A_2 \times \cdots \times A_n$ is called an n-ary relationship

② Binary Relation / 2-ary relation or

relation from $A$ to $B$

$R$, a relation, is a subset of $A \times B$.

ex $R$ on $\mathbb{R}$ to $\mathbb{R}$

$R = \{(a,b) \mid a \text{ is related to } b\}$

ex $R = \{(a,b) \mid a \cdot b = 1\}$

ex $(3, 1/3) \in R$ $\qquad$ $(0, x) \notin R$

$(-1,-1) \in R$

## Notation / Representation

- $R$ is a relation from $A$ to $B$
- $R$ is a subset of $A \times B$
- if $\boxed{R \text{ is a subset of } A \times A}$

  def: "$R$ on $A$"

$R: A \to B$

domain $\qquad$ codomain

- $(a,b) \in R$ $\quad$ <u>or</u> $\quad$ $aRb$ $\quad$ "a is related to b"

  $(a,b) \notin R$ $\quad$ <u>or</u> $\quad$ $a \not R b$ $\quad$ "a is <u>not</u> related to b"

- How many relationships?

  subsets of

  $|\text{subsets}| \Rightarrow |A \times B|$

$|\text{subsets}| = |P(A \times B)| = 2^{|A \times B|}$

set of all subsets

equiv relations?

ex $f: \mathbb{R} \to \mathbb{R}$

$f_1(x) = |x| = f_2 = \sqrt{x^2}$

## Representing R on set A or A to B

1. **List**: - $R = \{ (a_1, b_2), (a_2, b_1), \ldots \}$
   - Set builder

   propositional function

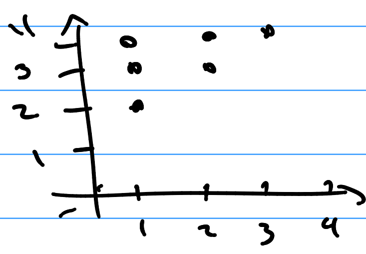
   $$R = \{ (a,b) \mid P(a,b) \}$$

   ex $R = \{ (a,b) \mid a < b \}$ on $A = \{1,2,3,4\}$

   $$R = \{ (1,2), (1,3), (1,4), (2,3), (2,4), (3,4) \}$$

2. **table**

| R | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | x | x | x |
| 2 | | | x | x |
| 3 | | | | x |
| 4 | | | | |




3. Zero-one matrix $M_R = [a_{ij}]$

$a_{ij} = 1 \quad (a_i, a_j) \in R$

$a_{ij} = 0 \quad (a_i, a_j) \notin R$

ex $M_R = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

## ④ directed graphs / digraphs

**R on set A**

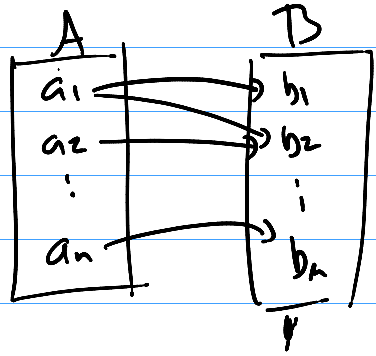
from arrow diagram

from domain to codomain

$R: A \to B$

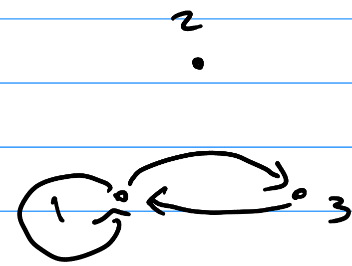
$(a_1, b_1) \in R$

$(a_1, b_2) \in R$




ex $R = \{(1,1), (1,3), (3,1)\}$ ←

on $A = \{1, 2, 3\}$

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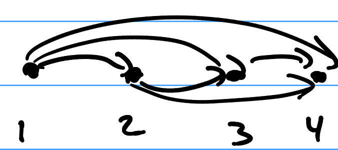
ex $R$ on $A = \{1, 2, 3, 4\}$, $R = \{(a,b) \mid a < b\}$

list: $R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

Matrix $M_R = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

digraph:

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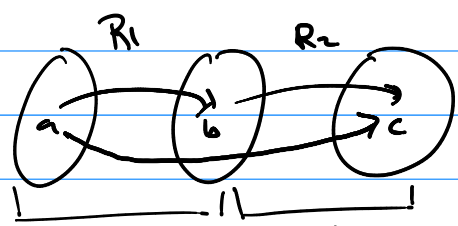
## Properties of Relations on set A

① operations

a) union $R_1 \cup R_2 = \{(a,b) \mid aR_1b \lor aR_2b\}$

5) Composition




using arrow diagram

$R_2 \circ R_1$

is $R_2(R_1(a)) = c$

$R_1(a) = b$ $\quad R_2(b) = c$ ← using functional notation

so $R_2(R_1(a)) = c$

$(R_2 \circ R_1)(a) = c$

## (2) Properties

a) **reflexive**. $R$ on set $A$ is called reflexive if $\forall e \, (e R e)$

(ex) $R$ on all dogs.

$R = \{ (a,b) \mid a \text{ and } b \text{ share a common parent} \}$

b) **irreflexive**. $\forall e \, (e \not{R} e)$

(ex)