

Math 322

Q's

transitive

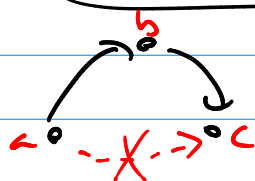
$$aRb \wedge bRc \rightarrow aRc$$

due Exam day Feb 12

$$7.1(1, 3, 7)$$

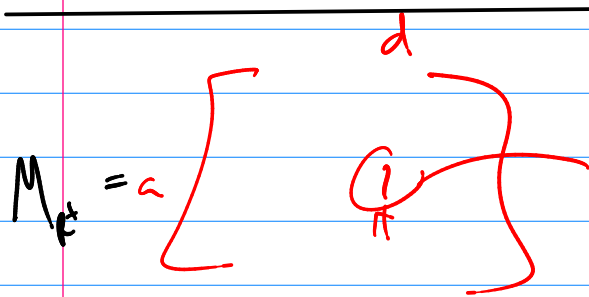
$$7.2(1, 2, 4, 7, 9)$$

$$7.3(1, 4, 8)$$



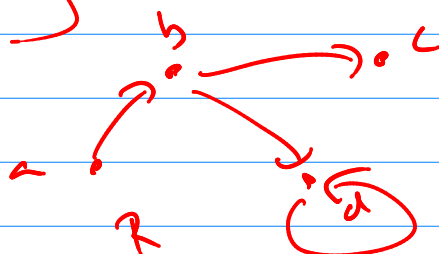
counter example to trans.

$$aRb \wedge bRc \nrightarrow aRc$$



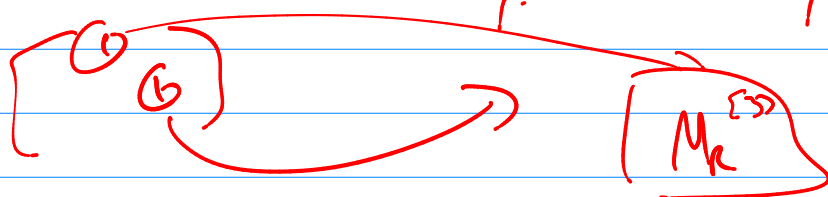
$$aR^+d$$

Verify a goes to d with a path of finite (non-zero) length



a, b, d is a path from a to d

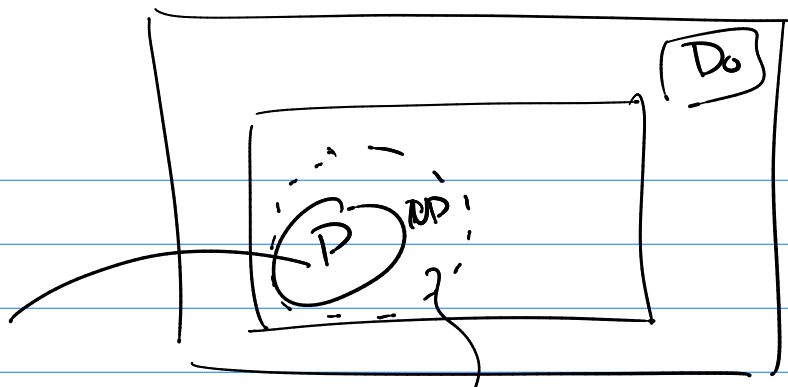
$$M_{R^+} = M_R \vee \underbrace{M_R^{\{2\}}}_I \vee \dots \vee \underbrace{M_R^{\{k\}}}_I$$



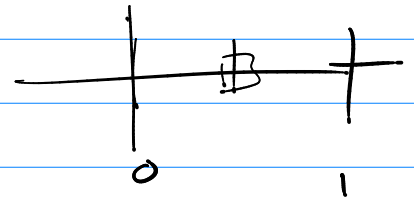
6.3 #1 $B = \{a, b\}$ $\mathcal{P}(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
 $\mathcal{R}_1 = \{(s, t) \mid s \leq t\}$ on $\mathcal{P}(B)$

7. $A = \{1, 2, 3, 6\} = \{1, 2, 3, 2 \cdot 3\}$

$$\mathcal{R}_2 = \{(s, t) \mid s \text{ is a factor of } t\}$$



3.14..



- check poly time check
- do $\rightarrow$ poly time

R is a relation..

check?

(1) ref.

(2) irref.

(3) sym.

(4) anti-sym

(5) as sym

(6) transitive

ch?

(7) Function

every $a \in A$ &
goes to exactly
one $b \in B$.

$f: A \rightarrow B$

$\exists (a) = b$

$a \in A, b \in B$

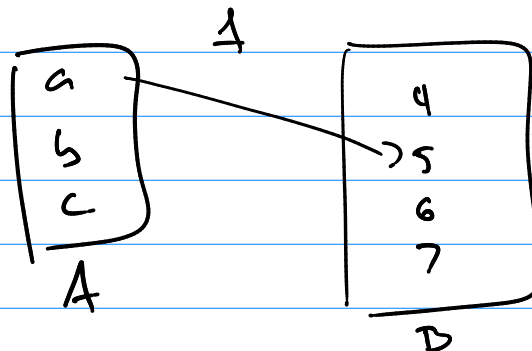
pre-image image

Def: a function from A to B $f: A \rightarrow B$

Def: set of all functions from A to B . B^A

(ex) $f: \{a, b, c\} \rightarrow \{4, 5, 6, 7\}$

ex:



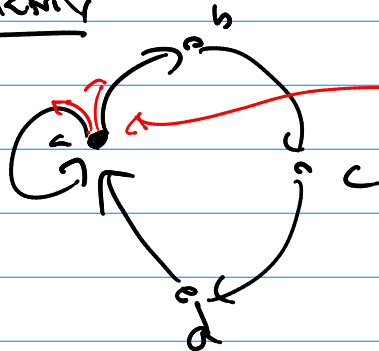
every one in
domain has
exactly one arrow

(ex) (continued)

$$\text{all } f: A \rightarrow B \quad |B^A| = 4 \cdot 4 \cdot 4 = 4^3 = |B|^{|A|}$$

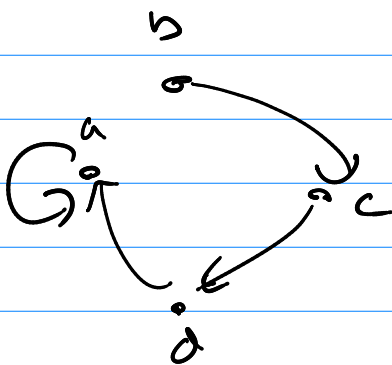
digraph / matrix

$$R = \{(a,a), (a,b), (b,c), (c,d), (d,a)\}$$



Not a function b/c $a \rightarrow \begin{matrix} a \\ b \end{matrix}$

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$



$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(I_3) a function

Subclasses of Functions

(1) one-to-one / injection

$$\forall a, b \in A \quad (\underline{f(a)} = \underline{f(b)} \rightarrow \underline{a=b})$$

$$\equiv \forall a, b \in A \quad (a \neq b \rightarrow f(a) \neq f(b))$$

(2) onto / surjection

$$\forall b \in B \quad \exists a \in A \quad f(a) = b$$

(range = codomain)

(3) both one-to-one and onto / bijection

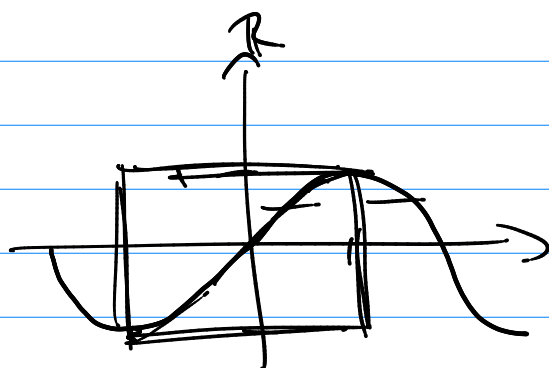
two applications & bijections

① Functional Composition.

$$(f \circ f^{-1})(x) = x$$

$$(f^{-1} \circ f)(x) = x$$

f^{-1} exists if and only if f is a bijection.



$$\begin{array}{ccc} [-\pi/2, \pi/2] & \rightarrow & [-1, 1] \\ A & \leftarrow & B \end{array}$$

$$\sin: A \rightarrow B$$

$y = \sin x \Rightarrow$ is variable for

② Cardinality

f a bijection exists for A to B

$$\text{then } |A| = |B|$$

bijection from $\{1, 2, 3, \dots\} \rightarrow S$

$$\aleph_0 = \underbrace{|\{1, 2, 3, \dots\}|}_{\text{not finite (infinite)}} = |S|$$

$\aleph_0$
1st transfinite
cardinals

we call S
countable

Note:

Cardinals "how much?"

$\hookrightarrow 0, 1, 2, 3, 4, 5, \dots, \aleph_0, \aleph_1, \aleph_2, \dots$
 Finite $\longleftrightarrow$ trans finite

$|\mathbb{Z}^+|$ $|\mathbb{R}| = |\mathcal{P}(\mathbb{Z}^+)| = 2^{\aleph_0}$
 $|\mathcal{P}(\mathbb{R})| = 2^{\aleph_1}$

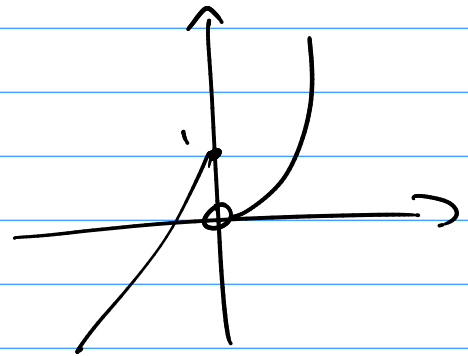
Functions to know (classic)

① piecewise defined / conditionally defined

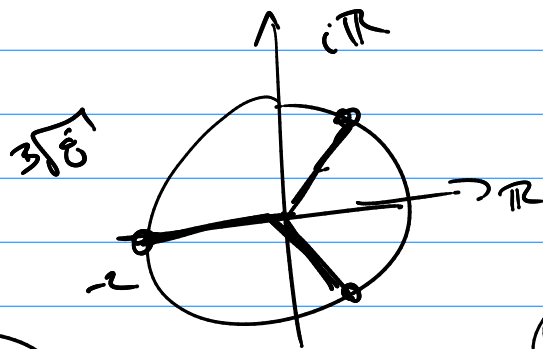
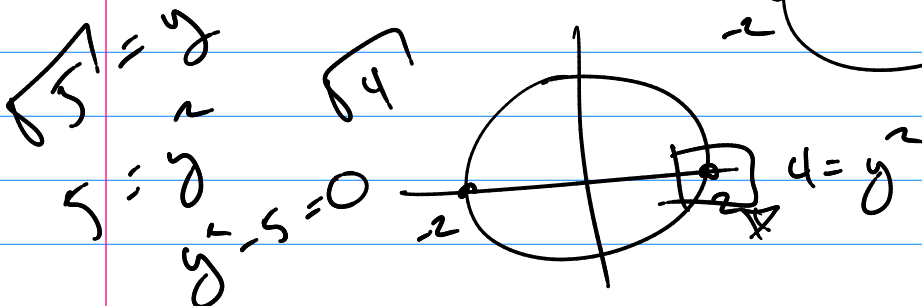
$$f(x) = \begin{cases} f_1(x) & \text{if } P_1(x) \\ f_2(x) & \text{if } P_2(x) \\ \vdots & \vdots \\ f_k(x) & \text{if } P_k(x) \end{cases}$$

logical functions

$$f(x) = \begin{cases} 2x+1 & x \leq 0 \\ x^2 & x > 0 \end{cases}$$



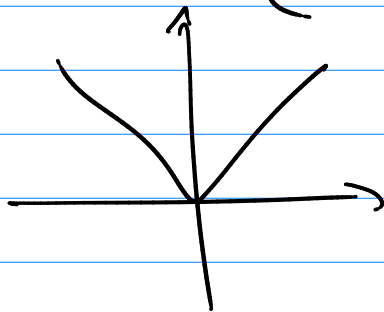
② $|x| = \sqrt{x^2}$



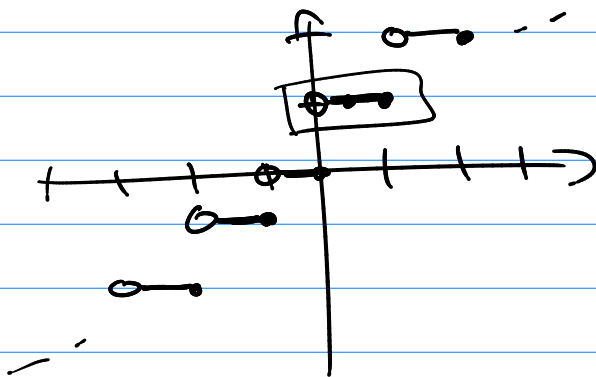
$\sqrt[3]{8} = 2$
 $\sqrt[3]{-8} = -2$

$\sqrt[3]{x^3} = x$
 $x = \sqrt[3]{x^3}$

$$|x| = \sqrt{x^2} = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

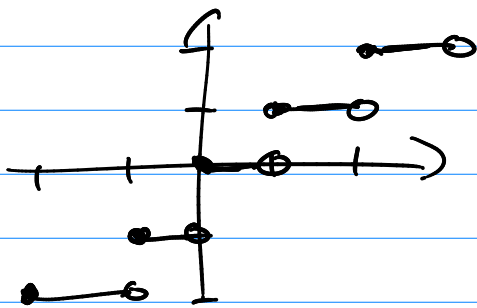


$$(3) \lceil x \rceil = \text{ceil}(x) = \begin{cases} 0 & \text{if } -1 < x \leq 0 \\ 1 & \text{if } 0 < x \leq 1 \\ 2 & \text{if } 1 < x \leq 2 \\ \vdots & \end{cases}$$



$$\text{ceil}(x) = k \quad \text{if } k-1 < x \leq k$$

$$(4) \lfloor x \rfloor = \text{floor}(x) = \begin{cases} k & \text{if } k \leq x < k+1 \end{cases}$$



$$\begin{cases} -1 & \text{if } -1 \leq x < 0 \\ 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < 2 \\ \vdots & \end{cases}$$