Math 322

Q's Due Fri 9.1 (2, 3, 5, 6, 7, 8)

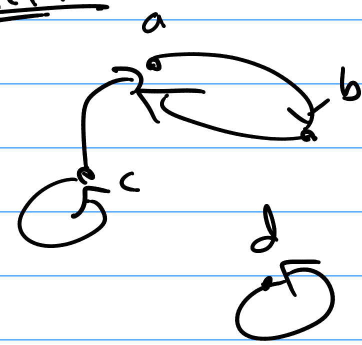
chapter 9 Graph theory

New toy!

Binary relation

digraphs

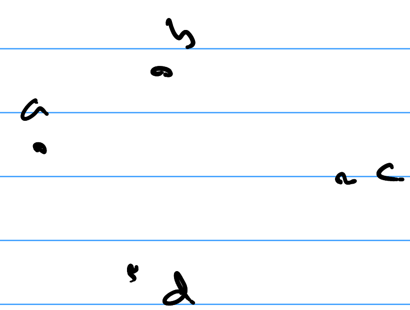
directed graph



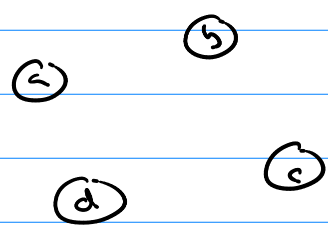
Def: G is a graph is a non-empty set of vertices, V, and a set of edges between those vertices, E.

$G = (V, E)$

vertices as dots



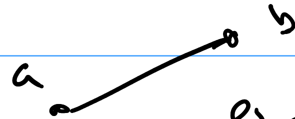
vs as circles



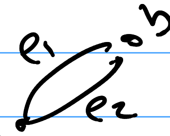
Graphs

Undirect graphs

E is made of pairs of vertices $e = \{a,b\}$

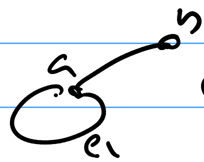


Def: a) multiple edge



$e_1 = \{a,b\}$ $e_2 = \{a,b\}$

b) loop

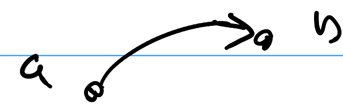


$e_1 = \{a,a\}$

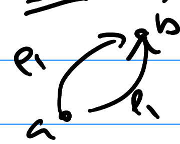
1. No loops, No Multiple edges
   → Call G a Simple Undirected Graph.
2. No loops, but multiple = OK
   → Call G a Undirected Multigraph
3. loops = OK, multiple = OK
   → call G a pseudograph.

directed graphs

E is made of ordered pairs $e = (a,b)$

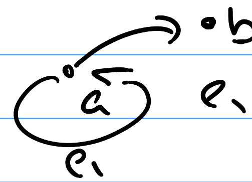


Def: a) multiple edge



$e_1 = (a,b)$
$e_2 = (a,b)$

b) loop



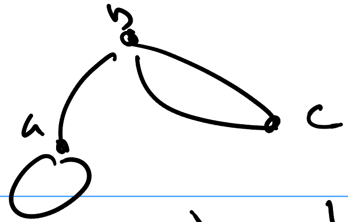
$e_1 = (a,a)$

loops are always OK.

1. No multi. edges
   → Call G a Simple directed graph
2. multiple = OK
   → call G a directed multigraph

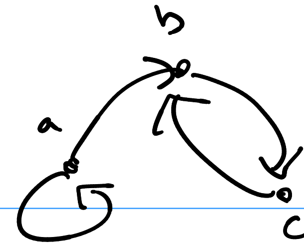
Mix undirected and directed edges
→ call G **Mixed**

ex



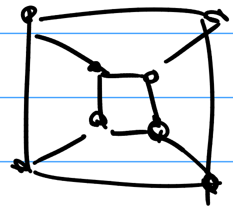
pseudograph

ex



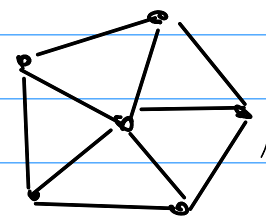
Simple directed graph

ex



Simple undirected graph

ex



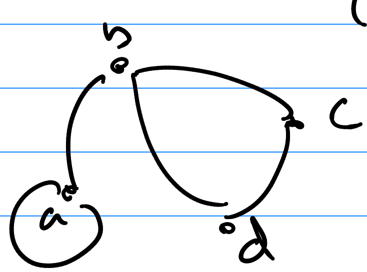
Simple undirected graph

observed properties $G = (V, E)$

1. $|V|$ = # of vertices
2. $|E|$ = # of edges
3. degrees

undirected

$\deg(v)$ = # of incident edges (except loops = 2)



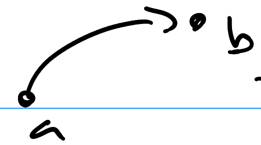
$\deg(a) = 3$ $\deg(c) = 2$

$\deg(b) = 3$ $\deg(d) = 2$

Handshake thm

$$\sum_{v \in V} \deg(v) = 2|E|$$

Directed Graphs



terminal vertices

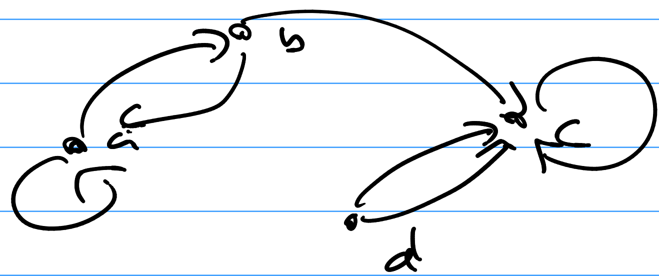
initial vertices

out degree

$\deg^+(v) = \#$ of edges with v as initial

in degree

$\deg^-(v) = \#$ of edges with v as terminal

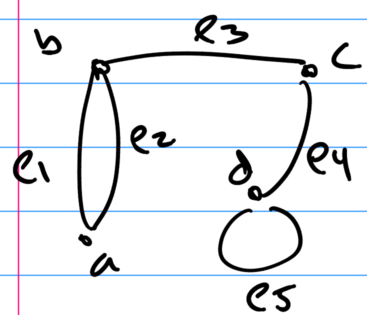


$|E| = 7$

$\deg^+(a) = 2$	$\deg^-(a) = 2$
$\deg^+(b) = 2$	$\deg^-(b) = 1$
$\deg^+(c) = 1$	$\deg^-(c) = 4$
$\deg^+(d) = 2$	$\deg^-(d) = 0$
Sum 7	7

thm $\sum_{v \in V} \deg^+(v) = \sum_{v \in V} \deg^-(v) = |E|$

Paths: seq of edges



path from a to d

$a : \overbrace{e_1, e_2, e_1, e_3, e_4, e_5}^{\text{Path}} : d$

length = 6

Notation if we do not have mult. edges in our path

(ex) $b : e_3, e_4, e_5, e_5, e_5 : d$ length = 5

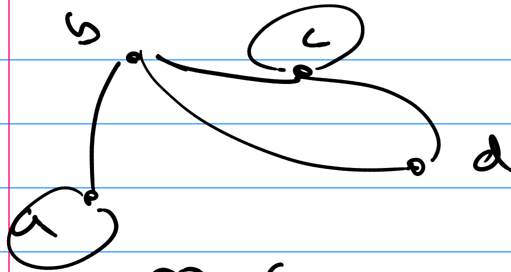
use vertices

b, c, d, d, d, d length = 5

Path names

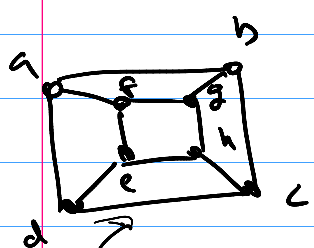
① Simple path never reuses an edge
② circuit starts and ends @ same vertex

ex

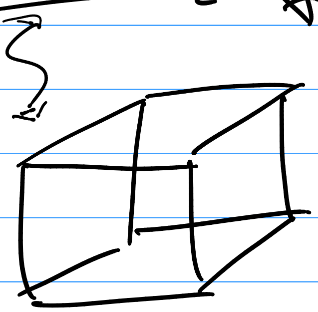


① circuit that is simple
b, c, d, b
simple circuit & length 3

② (not a path)
a, a, a, b, d, ~~b~~, a
no {d, a} !

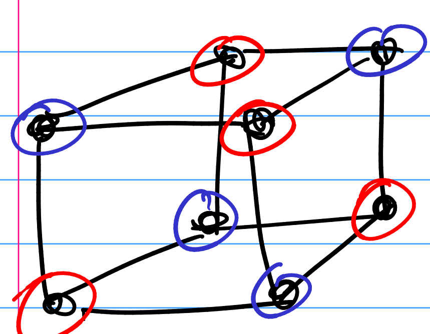


Simple undirected graph
$|V| = 8$
$|E| = 12$
$\forall v \; deg(v) = 3$



Same?

Bipartite if $V = V_1 \cup V_2$ can be partitioned into two disjoint sets such that edges only connect V_1 to V_2



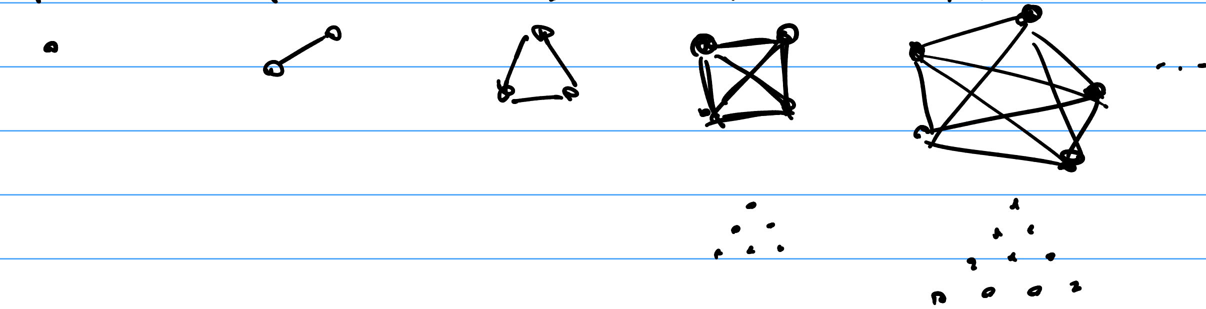
Coloring thm: color vertices with only two colors and edges always connect different colors
$\rightarrow G$ is bipartite

Simple undirected

Special Graphs

① Complete K_n

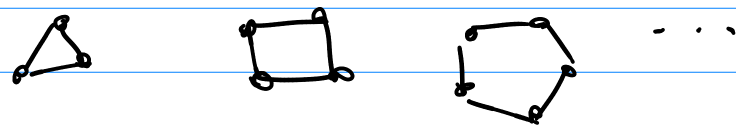
K_1 K_2 K_3 K_4 K_5



$|E| = 10$

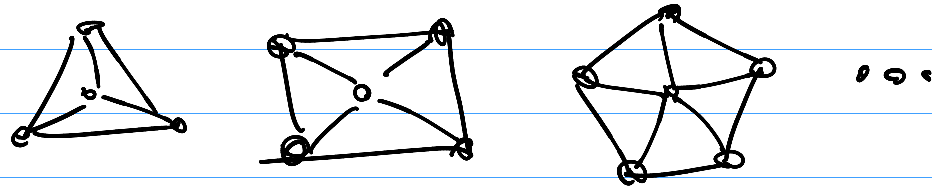
② Cycle $n \geq 3$

C_3 C_4 C_5

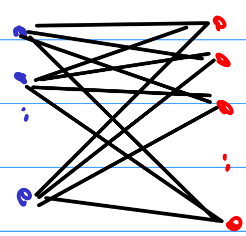


③ Wheel → C_n with an axle $|V| = n+1$

W_3 W_4 W_5



Complete bipartite $K_{n,m}$



n m