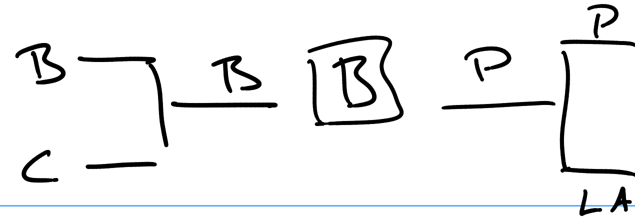
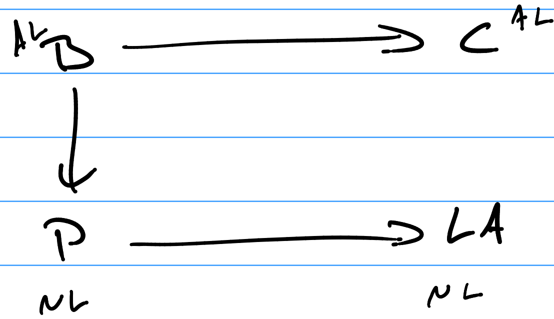
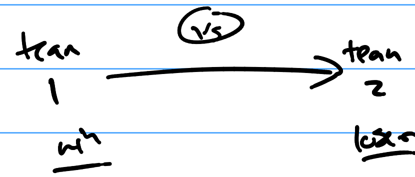
Math 322



Q's

1983 Baseball champ

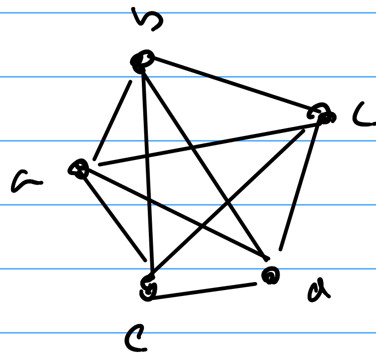




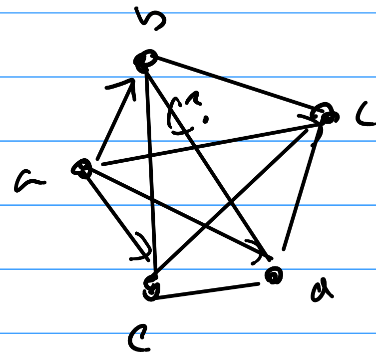
round robin

$|V| =$ teams

$|E| =$ games

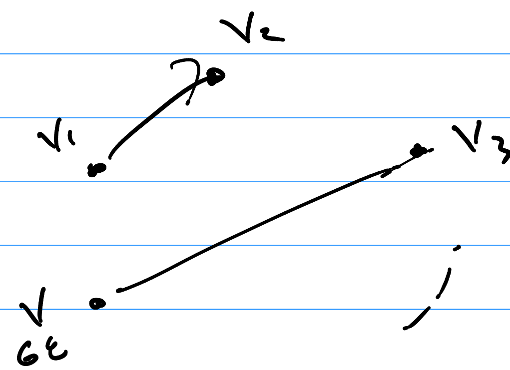


K_5



64 teams

single elim tournament

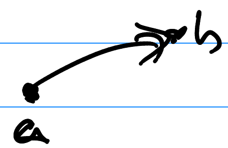


① 4 edges between 8 teams

$2 \cdot \binom{64}{2} \cdot \binom{66}{2}$

Representing Graphs

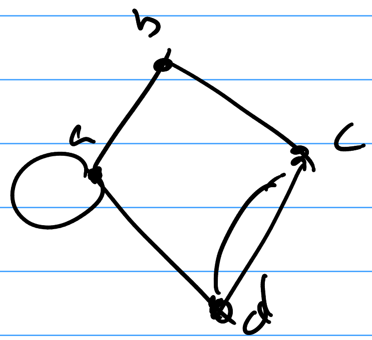
① draw them / lists $\quad G = (V, E)$



Computers

(2) Adjacency Matrix $A_G = [a_{ij}]$

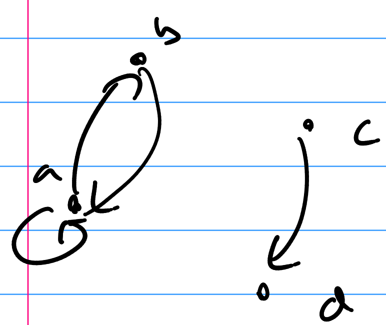
a_{ij} = # edges from v_i to v_j



$$A_G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

remember: $A_G^r = [c_{ij}]$ c_{ij} = # of paths from v_i to v_j

(3) Directory

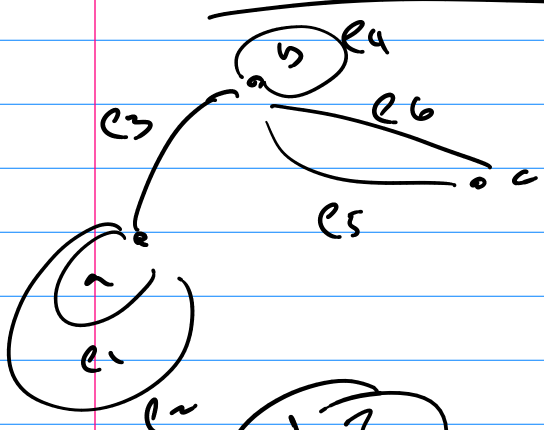


$G = \{ a:[a,b],\ b:[a],\ c:[d],\ d:[\,] \}$

(4) Incidence Matrix

$V = \{v_1, v_2, \ldots, v_n\}$

$E = \{e_1, e_2, \ldots, e_n\}$



$$I_G = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$|V| \times |E|$

why?

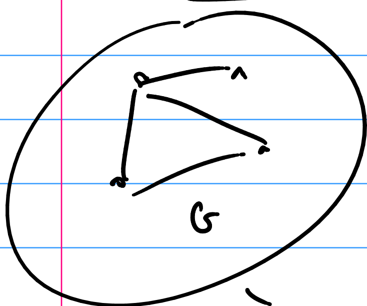
consider $\underset{|V| \times |E|}{I_G} \cdot \underset{|E| \times |V|}{I_G^T} = \underset{|V| \times |V|}{V_v}$

$$I_G^T \cdot I_G = F_E$$

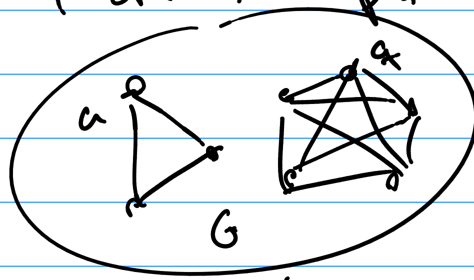
$|E| \times |V|$ $\quad$ $|V| \times |E|$ $\quad$ $|E| \times |E|$

Graph Features / Applications

① Connectivity (are there paths between vertices?)



connected



not connected

A_G^r

Undirected

- connected: path between any v_i, v_j

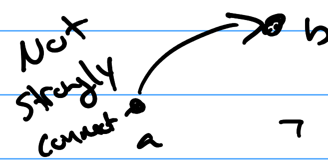
$$\boxed{A_{G^T} = A_G + A_G^2 + \cdots + A_G^{|V|}}$$

if there are any zeros.

→ not connected (undirected)

→ not strongly connected (directed)

Directed



path from a to b

$\neg$ path from b to a

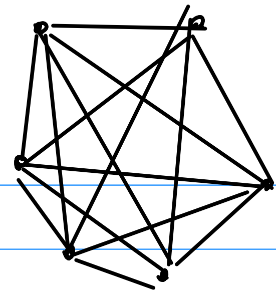
- strongly connected - path to and from any v_i, v_j

is strongly connected.

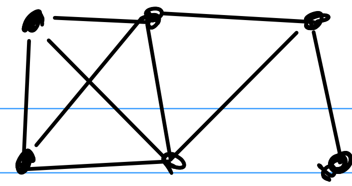
② Connected $\boxed{\text{but}}$ how robust?

does removal of vertices or edges break connectivity?

Ex



K_6



Vertex cut : set of vertices whose removal disconnects G

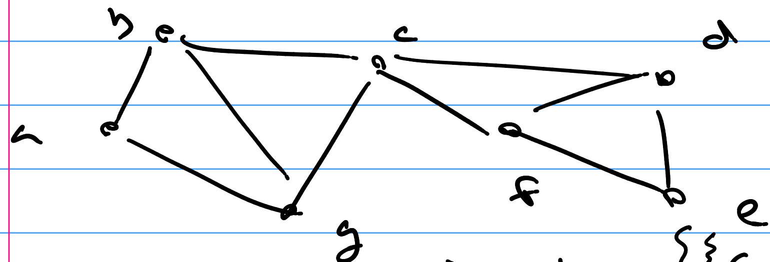
Edge cut : set of edges whose removal disconnects G

$\kappa(G)$ = min number of vertices for a vertex cut

$\lambda(G)$ = min number of edges for an edge cut

thm $\quad 1 \le \kappa(G) \le \lambda(G) \le \min_{v \in V} \deg(v) \le |V| - 1$

only for K_n



vertex cuts = $\{c\}, \{b,g\}, \{c,f\}$, etc

$\kappa(G) = 1$

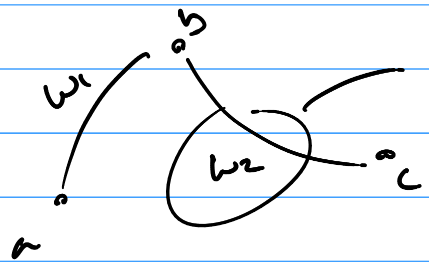
edge cuts = $\{\{c,d\}, \{c,f\}\}$,
$\{\{d,e\}, \{f,e\}\}$,
$\{\{b,g\}, \{b,c\}, \{a,g\}\}$, etc

$\lambda(G) = 2$

Applications: Paths

→ Weighted Graphs $G = (V, E)$

with a weight function $w(e)$ = weight of edge e



$w(\{b, c\}) = w_2$

length of path = sum of weights.

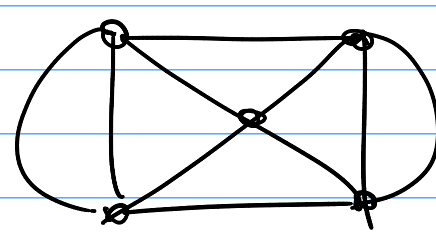
Note: old length def is simply $w(e) = 1$

① Connected ≡ existence of paths

② Special Paths

a) Euler Circuit: ① Simple circuit
② uses all edges

(ex) trace problem



b) Euler Path: ① Simple path (non-circuit)
② uses all edges

c) Hamilton Circuit ① Simple circuit
② uses all vertices exactly once

③ Minimal Paths for weighted graphs.