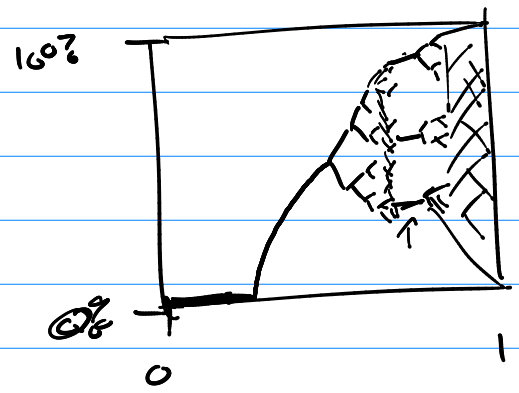
Math 322



Math = toys + rules ⇝ ch 11, ch 13

objects — operations — build to Boolean Algebra

1. Propositional logic ←
2. number theory
3. set theory
4. n-ary relations, binary relations
5. graph theory
6. elementary algebra
7. linear algebra

etc. (Maths)

unknown

Consider: Start — binary operation $\circ$ — = End

$\frac{1}{2} \cdot 2 \cdot x = \frac{1}{2} 4$

$1 \cdot x = 2$

$x = 2$

$Ax = C$

$A^{-1} A x = A^{-1} C$

$I x = A^{-1} C$

$x = A^{-1} C$

$f \circ g = h$ $\quad Sk(x) = 3$

$f(g(x)) = h(x)$ $\quad x = Sk^{-1}(3)$

$f^{-1}(f(g(x))) = f^{-1}(h(x))$

$g(x) = f^{-1}(h(x))$

Algebraic Systems / Mathematical Systems

(Math(s) = toys + rules)

① Non-empty set of elements, S

② Unary, binary operations

- unary operator is a function from S to S
- binary operator is a function from SxS to S

ex unary: $abs(-3) = 3$ $\quad sin(\pi/2) = 1$

binary: $2+3 = 5$ $\quad (2,3) \to 5$

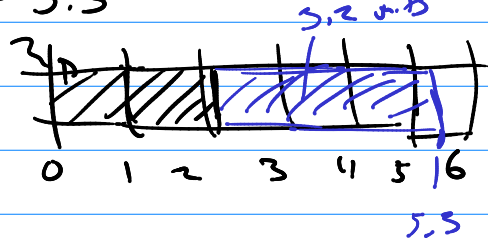
$2 \wedge 3 = 8$ $\quad (2,3) \to 8$

representing operators

① physical / inductive reasoning

ex: $2+3 = 5$ → 11, 111 (count = 5)

$7.1 + 3.2 = 5.3$



② ordered lists / tables if |S| is small

ex $|S| = 3$ $\quad S = \{a, b, c\}$

	a	b	c
a	b	c	a
b	c	a	b
c	a	b	c

Mayan

⊕	a	b	c
a	a	c	b
b	c	b	a
c	b	a	c

So a system $[S\ ;$ list operations $]$

(ex) grade 0 $[\{1,2,3,\ldots,100\}\ ;\ +\ ]$

(3) possible properties for S and it's operations

① closure $a * b \in S$

② commutative $a * b = b * a$

③ Associative $(e_1 \oplus e_2) \oplus e_3 = e_1 \oplus (e_2 \oplus e_3)$

④ Identity in S for operation, $e \equiv$ identity

⑤ Inverse for $a \in S$ such that $a \cdot (\ ) = e$ under operation $(\ ) \cdot a = e$

a's inv. $a^{(-1)}$ ← label of the word "inverse"

f^{-1} arcsin

$(e^x)^{-1}$ $e^{(-1)}$ ← label not power $\ln(x) = \log_e(x)$

⑥ Idempotent $a \oplus a = a$

⑦ Distributive (left-distrib / right-distrib)

↓ mix two binary ops.

(ex) ops: $\oplus_1, \oplus_2$

left distrib $e_1 \oplus_1 (e_2 \oplus_2 e_3)$

$(e_1 \oplus_1 e_2) \oplus_2 (e_1 \oplus_1 e_3)$

right distrib $(e_1 \oplus_1 e_2) \oplus_2 e_3$

$(e_1 \oplus_2 e_3) \oplus_1 (e_2 \oplus_2 e_3)$

(ex) $3(x+y) = 3x+3y$

$(x+y)3 = 3x+3y$

Distributive if operators have commutative prop.

(8) Involution for a unary operator.

(ex) $\sim a$ is a unary operator

if $\sim(\sim a) = a$ call unary operator, involute.

Levels of Abstraction

Concrete → Axiomatic → Universal

Specific Math(s)

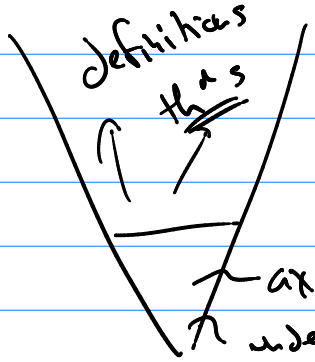
1. Arithmetic on Integers
2. Elementary Algebra
3. ⋮
4. etc

Axiomatic: consider 1 concrete example...

What is needed for it?

What are the fundamental objects fundamental truths

↑ axioms



Universal

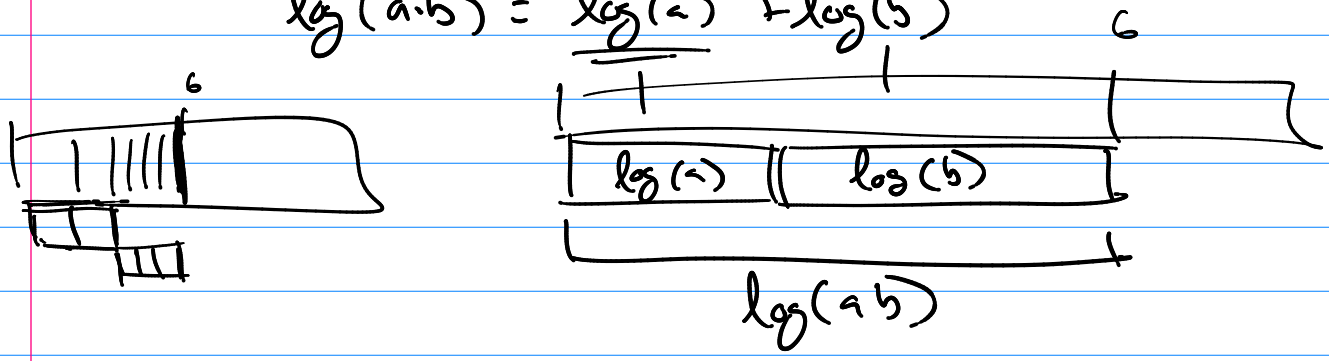
① Study concepts universal to all algebraic systems

② ⇝ isomorphic systems — $[\mathbb{R}^+, *]$ ⟷ $[\mathbb{R}^+, +]$

⇝ subsystems

etc

$$\log(a \cdot b) = \log(a) + \log(b)$$



Skirt Small : Monoid — M is a monoid if

$M = [V, *_1]$

V: elements; $*_1$: our binary operator

axioms and

① the identity element under $*_1$ exists in V

② Associative Property

$$a *_1 (b *_1 c) = (a *_1 b) *_1 c$$

↳ Make th^s for monoids → they apply to all systems that are monoids

th^n identity is unique

pf assume it isn't unique so e, f are identity. $e \neq f$

but $f = e \circ f = e$ so $\left(\begin{array}{l} e \neq f \\ \text{and} \\ e = f \end{array} \right) \equiv F$