Math 511

Systems of Linear Eqn's

$x_1 + 2x_2 - x_3 = 1$

$2x_1 - x_2 + x_3 = 3$

$-x_1 + 2x_2 + 3x_3 = 7$

Substitution ⇒

$x_3 = x_1 + 2x_2 - 1$

$3x_1 + x_2 = 4$

$2x_1 + 5x_2 = 10$

$x_2 = 4 - 3x_1$

$-11x_1 = -11$

$x_1 = 1 \quad x_2 = 1 \quad x_3 = 2$

$(1, 1, 2)$

Elimination:

$x_1 - 3x_2 = -5$

$2x_1 + x_2 = 4$

Elementary row ops ⇒ form systems with same soln. (Equivalent)

1. change row order
2. mult. row by a non-zero number
3. Mult. row + another row = New row
   - Ex: a · row 1 + row 2 = New Row 2

Goal of elimination is to use row ops to get a strict triangular form

ex $\begin{cases} 2x_1 - x_2 + x_3 = 1 \\ 4x_2 - x_3 = 2 \\ 3x_3 = 5 \end{cases}$ Strict triangular

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Matrices: Augmented Matrix representation of Lin. Sys.

ex $\begin{cases} 2x_1 - x_2 + x_3 = 1 \\ x_1 - x_2 + x_3 = 2 \\ 4x_1 + x_2 + 2x_3 = 1 \end{cases} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 4 & 1 & 2 & 1 \end{array}\right]$

Goal: use row ops to get to triangular forms

row echelon form

ex

$\left[\begin{array}{ccc|c} \boxed{2} & 1 & 3 & 1 \\ 1 & -1 & 4 & 2 \\ 3 & 0 & -1 & 7 \end{array}\right] \xrightarrow{-\frac{1}{2}r_1 + r_2 = N\,r_2} \left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 0 & -3/2 & 5/2 & 3/2 \\ 3 & 0 & -1 & 7 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 2 \\ 2 & 1 & 3 & 1 \\ 3 & 0 & -1 & 7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 4 & 2 \\ 0 & 3 & -5 & -3 \\ 0 & 3 & -13 & 1 \end{array}\right]$ $-2(1)+r_2 = N\,r_2$

Note: Aug. Matrix .. hints on Sols

$\left[\text{start system}\right] \xrightarrow{\text{row ops}} \left[\quad\right] \xrightarrow{\text{row ops}} \left[\quad\right] \rightarrow \cdots$

if ever get ..

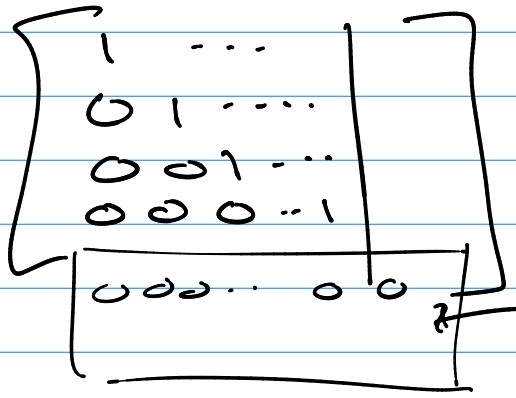
$$\left[\begin{array}{c|c} 0\ 0\ \cdots\ 0 & 1 \end{array}\right]$$

not zero

No Soln

if ever get

strict triangular



→ 1 Soln

not here or all zeros

$$\left[\begin{array}{ccc} 1 & 0 & 2 \\ 0 & 0 & 1 \end{array}\right]$$