Math 511

Elem. Algebra:

ex

$$3x + 4 = 3x(1+x)$$

$$3x + 4 = 3x + 3x^2$$

$$-3x + 3x + 4 = -3x + 3x + 3x^2$$

$$0 + 4 = 0 + 3x^2$$

Additive Inverse
Additive Identity

ex

$$2x = 5$$

$$\left(\tfrac{1}{2}\right)(2)x = \left(\tfrac{1}{2}\right)5$$

$$\boxed{1}\cdot x = \frac{5}{2}$$

$$x = 5/2$$

Multiplicative Inv.
Multiplicative Identity

---

toys: **Matrices:** A, $\underbrace{x, \vec{x}}_{\text{Vectors}}$

Rules: equality, $A+B$, A^T transpose (swap row/col)

ex

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad x = [1 \ 2 \ 3]^T$$

$$\alpha A = [\alpha a_{ij}]$$

AB?   Matrix · Matrix?

from systems of linear equations.

$$2x - 3y = 4$$

coef matrix $[2 \ \ -3]$ what about x? y? $[x \ \ y]^T$

Def:

$$\underset{1\times 2}{[2 \ \ -3]} \underset{2\times 1}{\begin{bmatrix} x \\ y \end{bmatrix}} = [4]$$

row vector · col vector = scalar

$A\mathbb{x}$

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & - & - & a_{mn} \end{bmatrix} = \begin{bmatrix} \vec{a_1} \\ \vec{a_2} \\ \vdots \\ \vec{a_m} \end{bmatrix} = [a_1 \ a_2 \cdots a_n]$$

row concept:

$$\underset{m\times n}{A}\ \underset{n\times 1}{\mathbb{x}} = \begin{bmatrix} \vec{a_1} \\ \vec{a_2} \\ \vdots \\ \vec{a_m} \end{bmatrix} \mathbb{x} = \begin{bmatrix} \vec{a_1}\mathbb{x} \\ \vec{a_2}\mathbb{x} \\ \vdots \\ \vec{a_m}\mathbb{x} \end{bmatrix}$$

col. concept

$$A\mathbb{x} = [a_1 \ a_2 \cdots a_n] \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{\mathbb{x}} = \underbrace{x_1 a_1 + x_2 a_2 + \cdots + x_n a_n}_{\text{linear combo of A's cols.}}$$

So, A system of linear eqn's

$$\text{is } \mathbb{A}\mathbb{x} = \mathbb{b} \quad \left[\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}\right.$$

$$\underset{m\times n}{A = [a_{ij}]} \qquad \underset{n \times 1}{\mathbb{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}} \qquad \underset{m \times 1}{\mathbb{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}}$$

So **Thm** $A\mathbb{x} = \mathbb{b}$ has a solution if and only if $\mathbb{b}$ can be written as a linear combo of A's col's.

Idea of proof

$$A\mathbb{x} = x_1 a_1 + x_2 a_2 + \cdots + x_n a_n$$

for $A\mathbb{x} = \mathbb{b}$ means $\boxed{x_1 a_1 + x_2 a_2 + \cdots + x_n a_n = \mathbb{b}}$

Matrix · Matrix?
$$\underset{m\times k}{A} \cdot \underset{k \times n}{B}$$

$$AB = A[\mathbb{b}_1 \ \mathbb{b}_2 \ \cdots \ \mathbb{b}_n]$$

$$= [A\mathbb{b}_1 \ A\mathbb{b}_2 \ \cdots \ A\mathbb{b}_n]$$

$$= \begin{bmatrix} \vec{a}_1 \mathbb{b}_1 & \vec{a}_1 \mathbb{b}_2 & \cdots & \vec{a}_1 \mathbb{b}_n \\ \vec{a}_2 \mathbb{b}_1 & \vdots & & \vdots \\ \vdots & & & \\ \vec{a}_m \mathbb{b}_1 & \vec{a}_m \mathbb{b}_2 & & \vec{a}_m \mathbb{b}_n \end{bmatrix} = [\vec{a}_i \mathbb{b}_j]$$

So $\underset{m\times k}{A}\underset{k\times n}{B} = [\vec{a}_i \, \mathbb{b}_j] = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{ik}b_{kj}$

$= \sum_{\ell=1}^{k} a_{i\ell} b_{\ell j}$

Algebra? (step 1 .. Laws)

Thm ① $A+B = B+A$

$A+B = [a_{ij} + b_{ij}] = [b_{ij} + a_{ij}] = B + A$

Note: $AB \neq BA$

are [not] always equal.

$\underset{2\times 3}{A}\ \underset{3\times 4}{B} = \underset{2\times 4}{C}$ $\cancel{\underset{3\times 4}{B}\ \underset{2\times 3}{A}}$

② $A(B+C) = AB + AC$

③ $(B+C)A = BA + CA$

④ $A+(B+C) = (A+B)+C$

⑤ $A(BC) = (AB)C$

⑥ $(\alpha\beta)A = \alpha(\beta A) = \beta(\alpha A)$

⑦ $\alpha(AB) = (\alpha A)B = A(\alpha B)$

⑧ $(\alpha+\beta)A = \alpha A + \beta A$

⑨ $\alpha(A+B) = \alpha A + \alpha B$

Def: $A^n = \underbrace{A \cdot A \cdot A \cdots A}_{n\text{-times}}$

(10) $(A^T)^T = A$

(11) $(\alpha A)^T = \alpha A^T$

(12) $(A+B)^T = A^T + B^T$

(13) $(AB)^T = B^T A^T$

Identity | Inv.

$A+B$: $0 = [0]$ all zero matrix | Add Inv of A is $(-1)A$

AB: $\underset{n \times n}{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ | Mult. Inv? ??

if it exists call it A^{-1}

and $AA^{-1} = I$

$A^{-1}A = I$