Math 511

Free Variables

system of eqns $\longrightarrow$ augmented matrix $\left[\quad\middle|\quad\right]$ $\longrightarrow$ elementary row ops

$\longrightarrow \left[\quad\middle|\quad\right] \longrightarrow \cdots \longrightarrow$
$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline \boxed{1} & 2 & 1 & 3 & 4 \\ 0 & 0 & \boxed{1} & 2 & 1 \\ 0 & 0 & 0 & \boxed{-3} & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

gauss elimination $\longrightarrow$ row eschelon form

x_2 is free (no lead)

x_1, x_3, x_4 have lead.

row ops to make zeros above lead $\longrightarrow \left[\quad\middle|\quad\right] \longrightarrow \cdots \longrightarrow$
$$\left[\begin{array}{cccc|c} \blacksquare & 2 & 0 & 0 & \\ 0 & 0 & \blacksquare & 0 & \\ 0 & 0 & 0 & \blacksquare & \\ 0 & 0 & 0 & 0 & \end{array}\right]$$

gauss-jordan

reduced row esch.

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline \boxed{1} & 2 & 1 & 3 & 4 \\ 0 & 0 & \boxed{1} & 2 & 1 \\ 0 & 0 & 0 & \boxed{-3} & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$x_2 =$ is free so $x_2 = \alpha$

$-3x_4 = 2 \qquad x_4 = -2/3$

$x_3 + 2x_4 = 1 \longrightarrow x_3 - 4/3 = 1 \rightarrow x_3 = 7/3$

$x_1 + 2x_2 + x_3 + 3x_4 = 4$

$x_1 + 2\alpha + 7/3 + (-2) = 4$

$x_1 = 11/3 - 2\alpha$

$x_1 = 11/3 - 2\alpha, \; x_2 = \alpha, \; x_3 = 7/3, \; x_4 = -2/3$

$(11/3 - 2\alpha, \; \alpha, \; 7/3, \; -2/3)$

$x_1 = 11/3 - 2\alpha, \; x_2 = 0 + \alpha, \; x_3 = 7/3 + 0\alpha, \; x_4 = -2/3 + 0\alpha$

$$\mathbb{X} = \begin{bmatrix} 11/3 \\ 0 \\ 7/3 \\ -4/3 \end{bmatrix} + \alpha \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

1.2 #9

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & 1 & \beta & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & \boxed{\beta-2} & 0 \end{array}\right]$$

$\beta = 2$ $\quad \beta - 2 = 0$ $\quad x_3$ is free! $\quad \infty$ Sol'n

$\beta \neq 2$ $\quad \beta - 2 \neq 0$ $\quad x_3$ is a lead so only 1 Sol'n (it is the trivial Sol'n)

Matrix Algebra ops $A + B$, αA, AB

Matrix Mult.

(1) $\vec{v}\,\vec{w} = v_1 w_1 + v_2 w_2 + \cdots + v_k w_k$

(2) $\underset{m\times n}{A}\ \underset{n\times 1}{\mathbb{X}} = \underbrace{\begin{bmatrix} \vec{a}_1 \mathbb{X} \\ \vec{a}_2 \mathbb{X} \\ \vdots \\ \vec{a}_m \mathbb{X} \end{bmatrix}} = \underbrace{x_1 a_1 + x_2 a_2 + \cdots + x_n a_n}$

(3) $\underset{m\times k}{A}\ \underset{k\times n}{B} = A[\mathbb{b}_1\ \mathbb{b}_2 \cdots \mathbb{b}_n] = [A\mathbb{b}_1\ A\mathbb{b}_2 \cdots A\mathbb{b}_n]$

$$= \begin{bmatrix} \vec{a}_1 \mathbb{b}_1 & \vec{a}_1 \mathbb{b}_2 & \cdots & \vec{a}_1 \mathbb{b}_n \\ \vdots & & & \\ \vec{a}_m \mathbb{b}_1 & \vec{a}_m \mathbb{b}_2 & \cdots & \vec{a}_m \mathbb{b}_n \end{bmatrix} = [\vec{a}_i\, \mathbb{b}_j]$$

Algebra: Laws: $A+B=B+A$

etc

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elem. Alg.

$2X+4=3$

Add Inv. $2X+4-4=3-4$

Add Iden. $2X+0=-1$

$2X=-1$

Mult. Inv. $\left(\frac{1}{2}\right)2X=\left(\frac{1}{2}\right)(-1)$

Mult. iden. $1\cdot X=-\frac{1}{2}$

$X=-\frac{1}{2}$

$(2)^{-1}$

Matrix Alg.

all zero

Add Identity is $O=[0]$

Add Inv. is $(-1)A$

Mult. Identity is $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

Mult. Inverse

$\Leftrightarrow$ is a matrix such that

$A\cdot(A^{-1})=I$

$(A^{-1})\cdot A=I$

A^{-1} is A's inv.

$$AB+C=D$$
$$AB+C+(-1)C=D+(-1)C$$
$$AB+O=D-C$$
$$AB=D-C$$
$$A^{-1}AB=A^{-1}(D-C)$$
$$I\cdot B=A^{-1}(D-C)$$
$$\boxed{B=A^{-1}(D-C)}$$

Note: $A^{-1}=\begin{bmatrix}1&2\\0&1\end{bmatrix}$ $D=\begin{bmatrix}3&0\\-1&1\end{bmatrix}$ $C=\begin{bmatrix}1&1\\1&1\end{bmatrix}$

$$B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)$$

$$B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -2 & 0 \end{bmatrix}$$

When A^{-1} exist?

Concept: "zero" (multiplicative dominator) is bad.

Elem. Alg. 0 has no inv. why?

b/c $a \cdot 0 = \boxed{0}$

$a \cdot 3 =$