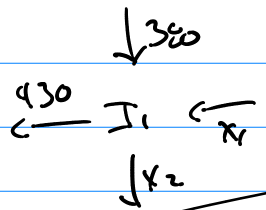
Math 511



ex

$$\begin{cases} x_1 + 38 = x_2 + 430 \\ 420 + 450 = x_1 + x_4 \\ x_2 + 540 = 420 + x_3 \\ 470 + x_3 = 400 + 420 \end{cases}$$

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = c_1$$

system of eqn's

Multiplicative Inv. A's inv. is noted as A^{-1}

and $A^{-1} A = A A^{-1} = I$

When does A^{-1} exist?

Need? ① A is $\underbrace{n \times n}_{\text{row} = \text{col}}$, A^{-1} is $n \times n$

terms: if A^{-1} exists
call A is non-singular, A is invertable

if A^{-1} does not exist
call A singular, A is not invertable

Goal ① find A^{-1}
② when does A^{-1} exist?

We will study solving systems of linear eqn's to do this...

→ do row ops to form equivalent systems

⇕

Find matrices that act as row ops to forms equivalent matrix equations.

[system of eqn's $\leadsto$ $A\mathbf{x} = \mathbf{b}$

$$2x + y = 1$$
$$x - 3y = 2$$

$A\mathbf{x} = \mathbf{b}$

$$\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 1 & -3 & 2 \end{array}\right] \iff \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Do?

row op 1 $\iff$ $E_1 A \mathbf{x} = E_1 \mathbf{b}$

gauss-elim

row op 2 $\iff$ $E_2 E_1 A \mathbf{x} = E_2 E_1 \mathbf{b}$

row op 3 $\iff$ $E_3 E_2 E_1 A \mathbf{x} = E_3 E_2 E_1 \mathbf{b}$

⋮

$$\left[\begin{array}{c|c} \text{row ech.} & \end{array}\right] \iff \underbrace{E_k \cdots E_3 E_2 E_1 A}_{U}\mathbf{x} = E_k \cdots E_1 \mathbf{b}$$

upper triangular

$$\boxed{U\mathbf{x} = \mathbf{c}}$$

Elementary Matrices

$\boxed{\text{Book}}$

E_{type1} $\cancel{E_1}$ E_2 E_3

vs E_1 E_2 E_3 E_4 ...

Type 1

(row swap)

row swap row i and row j

$E_{type1} = \begin{pmatrix} I \text{ with row } i \text{ and} \\ \text{row } j \text{ swapped} \end{pmatrix}$

ex
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

and $E_{type1}^{-1} = E_{type1}$

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type 2

$M \cdot$ row i = New row i

$E_{type2} = \begin{bmatrix} I \text{ with } M \\ \text{in } a_{ii} \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 2 & 6 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 1 & 3 & 4 \end{bmatrix}$$

$\frac{1}{2}6$

$E_{type2}^{-1} = \left[\underline{\underline{E_{type2} \text{ with } 1/M \text{ in } a_{ii}}} \right]$

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type 3

row i + M row j = New row i

$E_{type3} = \left[I \text{ and in } a_{ij} \text{ put } M \right]$

ex

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 0 & 0 & -5 \end{bmatrix}$$

want: row$_3$ + (-2) row$_1$ = New row$_3$

old r_3 + (-2) r_1 = New r_3

old r_3 = New r_3 + (2) r_1

$$E_{\text{type }3}^{-1} = E_{\text{type }3} \text{ with } -M \text{ @ } a_{ij}$$

Def: If $B = E_k \cdots E_3 E_2 E_1 A$

call B and A to be row equivalent

and we know any solution of $AX = B$

has same solution as $\boxed{E_k \cdots E_2 E_1 A} X = \boxed{E_k \cdots E_2 E_1 B}$

$\boxed{B} X = C$

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thm: the following are logically equivalent for A nxn

① A is non-singular (A^{-1} exists, A is invertible)

② $AX = 0$ (homogeneous system) has only trivial solution.

③ A and I are row equiv $\quad I = (E_k \cdots E_2 E_1) A$

So $(E_k \cdots E_2 E_1) = A^{-1}$

Note: $\underline{E_k \cdots E_2 E_1 I} = A^{-1}$

use this

$[A \mid I]$

$[E_1 A \mid E_1 I]$

$\vdots$

$[E_k \cdots E_2 E_1 A \mid E_k \cdots E_2 E_1 I]$

$\vdots$

$[I \mid A^{-1}]$

$[U \mid$

$U = E_k \cdots E_1 A$

$(E_1^{-1} \cdots E_k^{-1}) U = A$

$L\, U = A$