Math 511

$\boxed{\text{Q's}}$ $\boxed{1.4}$ $\to$ A^{-1} is a matrix such that

$$\underline{A^{-1}A} = \underline{A A^{-1}} = I$$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\quad A^{-1} = \frac{1}{a_{11}a_{22} - a_{21}a_{12}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

show true?

b/c $A(\alpha B) = \alpha AB$

Step 1 $\frac{1}{d}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \dots = \frac{1}{d}\begin{bmatrix} a_{11}a_{22} - a_{21}a_{12} & 0 \\ 0 & a_{11}a_{22} - a_{21}a_{12} \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Step 2 $\frac{1}{d}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \dots = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(a) $\underline{A^2 = 0}$ if $(I - A)$ is non-singular

show $(I - A)^{-1} = (I + A)$

Know: $(B)(B^{-1}) = I = (B^{-1})(B)$

Step 1 $(I - A)(I + A) = I^2 + IA - AI - A^2$
$= I + A - A - 0$
$= I + 0 - 0 = I$

Step 2 ?

Note: HW 2 Due Monday

Elem. Matrices

E_{type1}: swap row i, j

E_{type2}: M· row i = New row i

E_{type3}: row i + M row j = New row i

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 12 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 5/2 & -3/2 \\ 0 & 1 & 2 \end{bmatrix} \quad \underbrace{r_2 + (-3/2)\, r_1}_{} = N r_2$$

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gauss elim on augmented matrix $\left[\quad\middle|\quad\right]$ has exact Matrix Alg.

$$A x = b$$

row op $\qquad E_1 A x = E_1 b$

row op $\qquad \underline{E_2} E_1 A x = \underline{E_2} E_1 b$

$\vdots \qquad\qquad \vdots$

by product

Finished Gauss elim

$$\left[\,A \,\middle|\, \,\right]$$

row esch.

$$\boxed{E_k \cdots E_2 E_1 A}\, x = E_k \cdots E_1 b$$

So

$$E_k \cdots E_2 E_1 A = U$$

upper triangular

If I restrict myself to $\boxed{\text{only}}$ type 3 ops

$$\overbrace{\boxed{E_k \cdots E_2 E_1}}^{\text{type 3}} A = U$$

$$A = \overbrace{\boxed{E_1^{-1} E_2^{-1} \cdots E_k^{-1}}}^{\text{type 3}}\, U$$

but b/c we did gauss elim these are lower triangular as well.

Notice if I mult. these it is also lower triangular, call it L

$$\boxed{A = L\,U}$$

Note: check
$$\begin{bmatrix}1&0&0\\-2/3&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\-1/2&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&-2&1\end{bmatrix}\begin{bmatrix}1&-1&0\\\boxed{0}&2&1\\\boxed{0}&\boxed{0}&3\end{bmatrix}$$

$$E_3 E_2 E_1 A$$

Mult these at home and compare to the matrices themselves.

Partitioned Matrix

$$\underset{m\times n}{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & \cdots & & a_{mn} \end{bmatrix} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]$$

ex

$$A = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline -1 & 0 & 1 & 2 \\ 1 & 2 & 1 & 3 \end{array}\right]$$

$$\begin{array}{ccc|c} 0 & 1 & 0 & 0 \end{array}$$

$$\left[\begin{array}{c|c|c|c} A_{11} & A_{12} & A_{13} & A_{14} \\ \hline A_{21} & A_{22} & A_{23} & A_{24} \\ \hline A_{31} & A_{32} & A_{33} & A_{34} \end{array}\right] \qquad \left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \\ \hline B_{31} & B_{32} \\ \hline B_{41} & B_{42} \end{array}\right]$$

(partition) 3×4 (partition) 4×2

$$\boxed{A_{11}B_{11}} + A_{12}B_{21} + A_{13}B_{31} + A_{14}B_{41}$$

3×4

=

partition 3×2

When does A^{-1} exist?

aug. matrix

1×1 system of eq's $3x = 4 \rightarrow [3 \mid 4]$

Matrix alg $[3][x] = [4]$

$$\left[\tfrac{1}{3}\right][3][x] = \left[\tfrac{1}{3}\right][4]$$

$$[1]\,[x] = [4/3]$$

$$[x] = [4/3]$$

any prob?
$$[a][x] = [b]$$

$$[a]^{-1}[a][x] = [a]^{-1}[b]$$

$$\det([a]) = a$$

Notice: $\det(M) = 0$ inv. doesn't exist
$\det(M) \neq 0$ inv. does exist

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

use inv.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \boxed{\frac{1}{a_{11}a_{22} - a_{21}a_{12}}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right) = a_{11}a_{22} - a_{21}a_{12}$$

$\det(M) = 0 \rightarrow$ no inv.

$\det(M) \neq 0 \rightarrow$ has inv.