322

Partition Matrix — Add or Mult.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$\left[\begin{array}{c|c|c} A_{11} & A_{12} & A_{13} \\ \hline A_{21} & A_{22} & A_{23} \end{array}\right] + \left[\begin{array}{c|c|c} B_{11} & B_{12} & B_{13} \\ \hline B_{21} & B_{22} & B_{23} \end{array}\right]$$

$$= \left[\begin{array}{c|c|c} A_{11} + B_{11} & A_{12} + B_{12} & \cdots \\ \hline \cdots & \cdots & \cdots \end{array}\right]$$

1.5

12a)

$$AX + B = C$$

$$AX + B + (-1)B = C - B$$

$$AX + 0 = C - B$$

$$\boxed{AX = C - B}$$

we can find A^{-1}

$$[A \mid I] \rightarrow [I \mid A^{-1}]$$

$$A^{-1}AX = A^{-1}(C - B)$$

$$IX = A^{-1}(C - B)$$

$$X = \boxed{(A^{-1})(C - B)}$$

$$AX + B = C$$

$$X = \frac{C - B}{A}$$

-10

does A^{-1} exist?

$\boxed{\text{th}^{m}}$ ① A is singular if and only if $\det(A) = 0$

② A is non-singular iff $\det(A) \neq 0$

From chapter 1 we also have

Singular	vs	Non-singular
① A is singular		① A is non-singular
② $\det(A) = 0$		② $\det(A) \neq 0$
③ A^{-1} does not exist		③ A^{-1} does exist
④ A is not invertable		④ A is invertable
⑤ $A\mathbf{x} = \mathbf{0}$ ← all zero col. vector has a non-trivial soln		⑤ $A\mathbf{x} = \mathbf{0}$ has only trivial soln
⑥ A is not row equiv to I		⑥ A is row equiv. to I

$$[A \mid I] \xrightarrow[\text{ops}]{\text{row}} [I \mid A^{-1}]$$

Note: if you want to find $A^{-1}B$

use $[A \mid B] \underset{\text{ops}}{\overset{\text{row}}{\rightsquigarrow}} [I \mid A^{-1}B]$

det(A) = ?

1. $\det([a]) = a$
2. $\det\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right) = a_{11}a_{22} - a_{21}a_{12}$
3. $\det(A)$ with $A_{n \times n}$?

Notation: 1. Minor M_{ij} is A with row i, col j removed

ex $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 1 \\ 4 & -1 & 2 \end{bmatrix}$ $M_{23} = \begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$

2. cofactor $A_{ij} = (-1)^{i+j} |M_{ij}|$

forms a sign matrix $\begin{bmatrix} + & - & + & - & \cdots \\ - & + & - & + & \cdots \\ + & - & + & - & \cdots \\ \vdots \end{bmatrix}$

det(A) by cofactor expansion.

1. pick a row or a column (pick the one with the most zeros or at least smallest numbers)

ex (2) pick row k

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & & a_{1n} \\ a_{21} & a_{22} & \cdots & & a_{2n} \\ a_{k1} & a_{k2} & a_{k3} & \cdots & a_{kn} \\ \vdots \\ a_{n1} & \cdots & & & a_{nn} \end{bmatrix}$$

$$\det(A) = a_{k1}A_{k1} + a_{k2}A_{k2} + \cdots + a_{kn}A_{kn}$$

or pick col. k

$$\det(A) = a_{1k}A_{1k} + a_{2k}A_{2k} + \cdots + a_{nk}A_{nk}$$

$$\det\begin{pmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 4 \end{pmatrix} = \cancel{0A_{21}} + 1A_{22} + \cancel{0A_{23}}$$

$$= (-1)^{2+2}\, 1 \begin{vmatrix} -1 & 3 \\ 2 & 4 \end{vmatrix}$$

$$= -10$$

1st row

$$= -1\begin{vmatrix} 1 & 0 \\ 1 & 4 \end{vmatrix} - 2\begin{vmatrix} 0 & 0 \\ 2 & 4 \end{vmatrix} + 3\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}$$

(3) det(triangular matrix)

- upper triangular
- lower triangular
- diagonal

ex

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 1\begin{vmatrix} 4 & 1 \\ 0 & 2 \end{vmatrix} + \cancel{0\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix}} + \cancel{0\begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}}$$

$$= \begin{vmatrix} 4 & 1 \\ 0 & 2 \end{vmatrix} = 4 \cdot 2 = 1 \cdot 4 \cdot 2$$

so $\det\left(\begin{bmatrix} t_{11} & & & \\ & t_{22} & & \\ & & \ddots & \\ 0 & & & t_{nn} \end{bmatrix}\right) = t_{11} t_{22} \cdots t_{nn}$

upper triangular

$\boxed{\det(A) = |A|}$ notation

So $\left|\begin{bmatrix} -1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}\right| = (-1)(1)(2)(\tfrac{1}{2}) = \boxed{-1}$

would be nice to always have $\det(U)$!

But gauss elim $\boxed{U} = \underline{E_k} \cdots \underline{E_2}\,\underline{E_1} \boxed{A}$ (row equiv. matrices)

want $\det(U) = \det(E_k \cdots E_2 E_1 A)$

① $\det(EA) = \det(E)\det(A)$

② $\det(E_{\text{type}1}) = -1$

③ $\det(E_{\text{type}2}) = m$

④ $\det(E_{\text{type}3}) = 1$

$\begin{bmatrix} 1 & & \\ & \ddots & \\ m & & 1 \end{bmatrix}$

det(A) by row ops. (gauss elim)

$$\det(U) = \boxed{\det(E_k) \cdots \det(E_2)\det(E_1)}\,\det(A)$$

$$\det(A) = \frac{\det(U)}{\det(E_k) \cdots \det(E_1)}$$