Math 511

$\rightarrow X = \{x_{ij}\} = \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vdots \\ \vec{x}_n \end{bmatrix} = [\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_r]$

Q 1.6 (8)

$\underset{n\times n}{A}\ \underset{n\times r}{X} = \underset{n\times r}{B}$ iff $A\mathbf{x}_j = \mathbf{b}_j \quad j = 1,2,\ldots,r$

$\rightarrow B = \{b_{ij}\} = \begin{bmatrix} \vec{b}_1 \\ \vdots \\ \vec{b}_n \end{bmatrix} = [\mathbf{b}_1 \cdots \mathbf{b}_r]$

$A\mathbf{x}_1 = \mathbf{b}_1$
$A\mathbf{x}_2 = \mathbf{b}_2$
$A\mathbf{x}_3 = \mathbf{b}_3$
$\vdots$
$A\mathbf{x}_r = \mathbf{b}_r$

$\underline{AX = B}$ iff $A[\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_r] = [\mathbf{b}_1\ \mathbf{b}_2 \cdots \mathbf{b}_r]$

iff $[A\mathbf{x}_1\ A\mathbf{x}_2 \cdots A\mathbf{x}_r] = [\mathbf{b}_1\ \mathbf{b}_2 \cdots \mathbf{b}_r]$

iff $A\mathbf{x}_1 = \mathbf{b}_1$
and $A\mathbf{x}_2 = \mathbf{b}_2$
$\vdots$
and $A\mathbf{x}_r = \mathbf{b}_r$

iff $\underline{A\mathbf{x}_i = \mathbf{b}_i \quad i = 1,2,\ldots,r}$

$\underbrace{(E_k \cdots E_2 E_1)}_{A^{-1}} A = I$

$[A \mid I]$
$\rightarrow [E_1 A \mid E_1 I]$
$\rightarrow [E_2 E_1 A \mid \underline{E_2 E_1 I}]$

$E_7 E_6 E_5 E_4 E_3 E_2 E_1$

on factoring

elem. algebra

$$2x^2 + 10x + 0$$

$$(2x+1)x$$

ex

$$AX + X = \cancel{AX + X \cdot I}$$

vs

$$A \cdot X + I \cdot X = (A + I)X$$

ex

$$AX + X = CX + D$$

$$AX + X - CX = D$$

$$AX + IX - CX = D$$

$$(A + I - C)X = D$$

$$X = (A + I - C)^{-1} D$$

ex

$$ST - S + E = D + SQ$$

$S = ?$

$$ST - SI - SQ = D - E$$

$$S(T - I - Q) = (D - E)$$

$$\boxed{S = (D-E)(T-I-Q)^{-1}}$$

$A^{-1}C$

$$AB = C$$

Matlab or Octave

$[A|C]$

$\vdots$

$[I|A^{-1}C]$

$$B = A^{-1}C$$

$B = inv(A) * C$

$= A \backslash C$

$$A = C B^{-1}$$

$A = C * inv(B)$

C / B

.1 $\to$ 2^{16} double

$\to$ $2^{1?}$ subtract 1

Notation: $\det(A) = |A|$

cofactors

elimination

A is 5x5 ~ 200 flops ~ 44

A is 10x10 ~ 6 million flops ~ 300

2.3 adjoint of A

cofactors

$$A_{ij} = (-1)^{i+j}|M_{ij}|$$

by row k

$$\det(A) = a_{k1}A_{k1} + a_{k2}A_{k2} + \dots + a_{kn}A_{kn}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{k1} & a_{k2} & \cdots & a_{kn} \\ \vdots & & & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix}$$

row i of A $\quad [a_{i1} \; a_{i2} \; \cdots \; a_{in}]$

vs cofactors of row j $\quad [A_{j1} \; A_{j2} \; \cdots \; A_{jn}]$

thm

$$a_{i1}A_{j1} + a_{i2}A_{j2} + \cdots + a_{in}A_{jn} = \begin{cases} \det(A) & i=j \\ 0 & i \neq j \end{cases}$$

$$\vec{a}_i \begin{bmatrix} A_{j1} \\ A_{j2} \\ \vdots \\ A_{jn} \end{bmatrix}$$

So to find $\det(A)$ by cofactors .. we calculate cofactors

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix} \quad \text{vs} \quad \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & & \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}$$

so

$$A \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ \vdots & & & \\ A_{n1} & \cdots & \cdots & A_{nn} \end{bmatrix}^T$$

$$= \begin{bmatrix} \vec{a}_1 \\ \vdots \\ \vec{a}_n \end{bmatrix} \begin{bmatrix} A_{11} & \cdots & A_{n1} \\ A_{12} & & \vdots \\ \vdots & & \vdots \\ A_{1n} & & A_{nn} \end{bmatrix} = \begin{bmatrix} \det(A) & 0 & \cdots & 0 \\ 0 & \det(A) & \cdots & \\ & & \ddots & \\ 0 & & & \det(A) \end{bmatrix}$$

ok

$$A \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & & \\ A_{n1} & \cdots & \cdots & A_{nn} \end{bmatrix}^T = \det(A)\, I$$

$$(A)\left(\frac{1}{\det(A)}\underbrace{\begin{bmatrix} A_{11} & A_{12} \cdots & A_{1n} \\ \vdots & & \\ A_{n1} & \cdots & A_{nn} \end{bmatrix}^T}_{\text{adjoint of } A}\right) = I$$

$$\boxed{A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)}$$