Math 511

Ch 3 Vector Spaces

Sets: $S = \{$ unordered collection of elements $\}$

(ex) $S = \{0, 1, 2, 3, 4\}$

$S = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$S = \{e \mid e$ is an integer and 2 divides $e\}$

($\mid$ = such that)

Inductively define them

1. Basis → given a basic element
2. Inductive Step → explains how to create new elements.

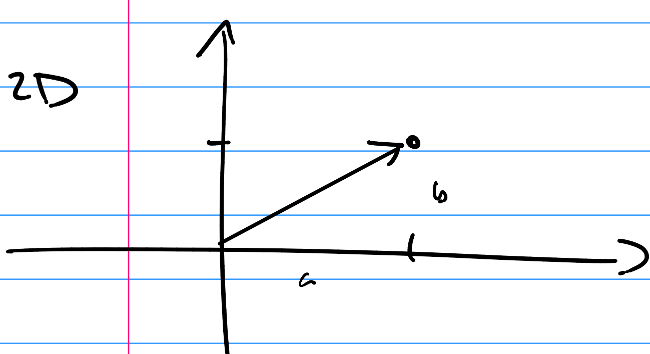
(ex) ① $3 \in S$ (element of)

② if e_1, e_2 are in S then $e_1 + e_2 \in S$

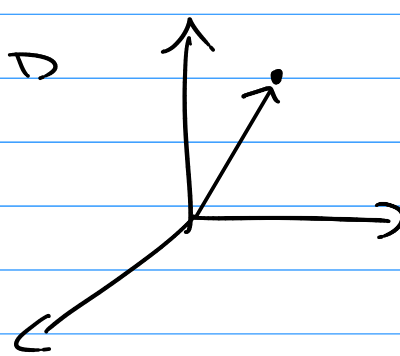
So $S = \{3, 6, 9, 12, 15, \ldots\}$

Vector Space (eventually we get to "Vector" Space)

2D



3D

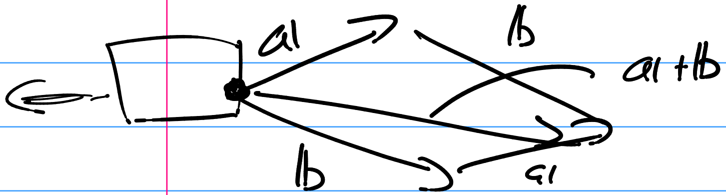


basic element: **Vector**

Vector Space: $\mathbf{a}, \mathbf{b}$ are vectors in V

Define: two operators ① $\alpha \mathbf{a} = [\alpha a_1 \ \alpha a_2 \cdots \alpha a_n]^T$

② $\mathbf{a} + \mathbf{b} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}$



Fund. Properties of $\mathbf{a}, \mathbf{b} \in V$, $\alpha \mathbf{a}$, $\mathbf{a} + \mathbf{b}$
$\mathbf{c} \in V$

Axioms **closure:** (1) $\mathbf{a} + \mathbf{b} \in V$

(2) $\alpha \mathbf{a} \in V$

Properties of $\mathbf{a}+\mathbf{b}$

A1) $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

→ A2) $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$

A3) there is a zero vector $\mathbf{0}$, $\mathbf{a} + \mathbf{0} = \mathbf{a}$

A4) there is an add. inv. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$.

Properties of $\alpha \mathbf{a}$

A5) $\alpha(\mathbf{a} + \mathbf{b}) = \alpha \mathbf{a} + \alpha \mathbf{b}$

A6) $(\alpha + \beta)\mathbf{a} = \alpha \mathbf{a} + \beta \mathbf{a}$

A7) $(\alpha \beta)\mathbf{a} = \alpha(\beta \mathbf{a}) = \beta(\alpha \mathbf{a})$

A8) $1\mathbf{a} = \mathbf{a}$

Note: Corollaries to above ← truths that follow above

$\boxed{\text{th}^m}$

① $0\mathbf{a} = \mathbf{0}$

② if $\mathbf{a} + \mathbf{b} = \mathbf{0}$ then $\mathbf{b} = -\mathbf{a}$

③ $(-1)\mathbf{a} = -\mathbf{a}$

So if we choose "strange" elements (non-typical vectors (2D/3D))

ex ① e is n^{th}-dimensional vector $\mathbb{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

② e is a n-term polynomial

$$\mathbb{V} = a_1 + a_2 x + a_3 x^2 + a_4 x^3 + \cdots + a_n x^{n-1}$$

③ e is a continuous function

$$\mathbb{V} = f(x) \quad \text{where } f(x) \text{ is cont. over } [a, b]$$

④ e is a $m \times n$ matrix

$$\mathbb{V} = [a_{ij}] = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

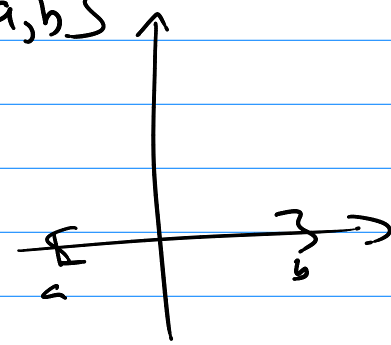
and for each we need to define..

$\alpha \mathbb{a}$? $\mathbb{a} + \mathbb{b}$?

then check the 10 axioms.

→ if true .. call the set a Vector Space

(iv) $C[a,b]$ all cont. functions over $[a,b]$



$f, g \in C[a,b]$

def: $(\alpha f)(x) = \alpha f(x)$

$(f+g)(x) = f(x) + g(x)$