Math 511

Q's → HW Due Wed

Vector Space $\ell_1, \ell_2, \ell_3 \in V$

define: $\alpha \ell$

$\ell_1 + \ell_2$

3.1 #13 $\ell \in \mathbb{R}$ ← real numbers

normal scalar mult. $\alpha\ell = \alpha \cdot \ell$

ex $(3 \cdot 2 = 6)$

$\ell_1 \oplus \ell_2 = \max(\ell_1, \ell_2)$

Vector Space? check all 10 axioms.

closure: (1) check is $\ell_1 + \ell_2$ still a real?

max(real, real) is real true!

(2) $\alpha \ell_1$ = real · real = real? true!

$\ell_1 \oplus \ell_2$ axioms
- A1) Commutative?
- A2) assoc?
- A3) zero object? $\boxed{\ell \oplus 0} = \ell$
- ~~A4)~~

$\max(\ell, 0) = \ell$

?

$\alpha \ell_1$ axioms
- A5)
- A6)
- A7)
- A8)

0 1 0 assume 0 exists (means max(any number, 0) = any number)

ok how about $\max(0-1, 0) = 0$

3.2 Subspace

Def: Subspace is

① a subset of a Vector Space

② it is also a Vector space.

but we don't need to check all axioms (most are fine b/c αv_1, $v_1 + v_2$ is known from the larger Vector Space)

Only need 3 things

closure: ① $v_1, v_2 \in S$ then $\alpha v_1 \in S$

② $v_1 + v_2 \in S$

③ $0 \in S$

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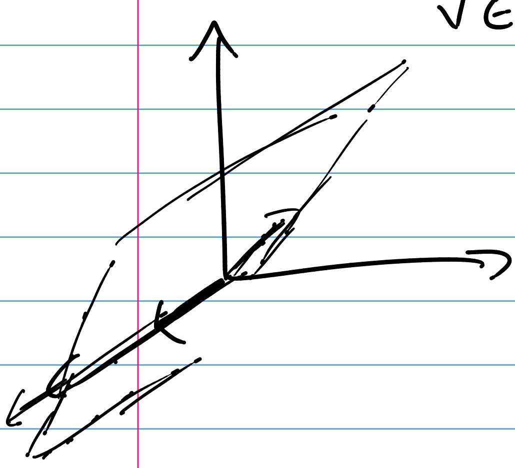
ex $V = \mathbb{R}^3$ (3D space)

$$S = \left\{ \begin{bmatrix} a \\ b \\ b \end{bmatrix} \,\middle|\, a, b \text{ are real} \right\}$$

ex $\begin{bmatrix} 2 \\ \pi \\ \pi \end{bmatrix} \in S$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \in S$ etc

$v \in S$ is $\begin{bmatrix} a \\ b \\ b \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$



① is $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ in S yes!

② is $\alpha \begin{bmatrix} a \\ b \\ b \end{bmatrix} \in S$

$\begin{bmatrix} \alpha a \\ \alpha b \\ \alpha b \end{bmatrix} \in S$ yes!

③ $\begin{bmatrix} a \\ b \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \\ d \end{bmatrix} = \begin{bmatrix} a+c \\ b+d \\ b+d \end{bmatrix} \in S$ yes!

So S $\boxed{\text{is}}$ a subspace

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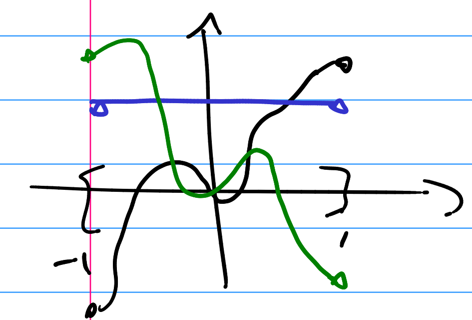
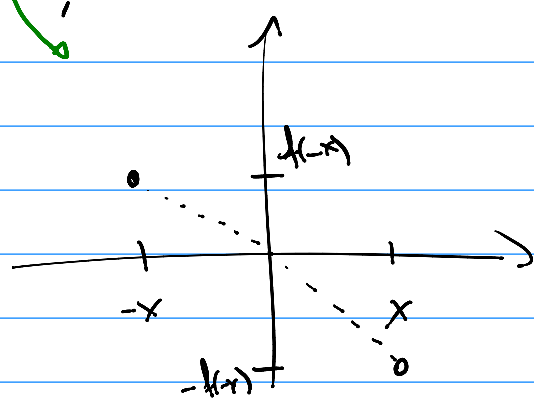
ex $C[-1,1]$ Vector space of continuous functions over $[-1,1]$



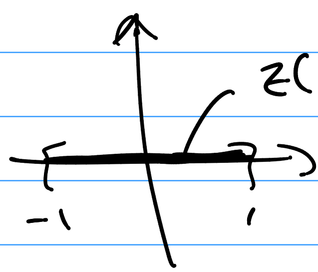
Subset is all the odd functions.

$f(-x) = -f(x)$

or symmetric about origin.



① is 0 in S? $0 \xrightarrow{?} \boxed{z(x) = 0}$ is odd



$$z(-x) = 0$$
$$z(x) = 0$$
$$-z(x) = 0$$

② $(\alpha f)(x) = \alpha f(x)$

if $f(x)$ is odd is $\alpha f(x)$ odd?

$$\alpha f(-x) \underset{f}{=} \alpha(-f(x)) = -\alpha f(x)$$

$\boxed{\text{true}}$ is odd!

3) check if f, g are odd

→ then $(f+g)(x) = f(x) + g(x)$ is still odd

(it is true)

So $S = \{$ all odd functions $\}$ $\boxed{\text{is}}$ a subspace

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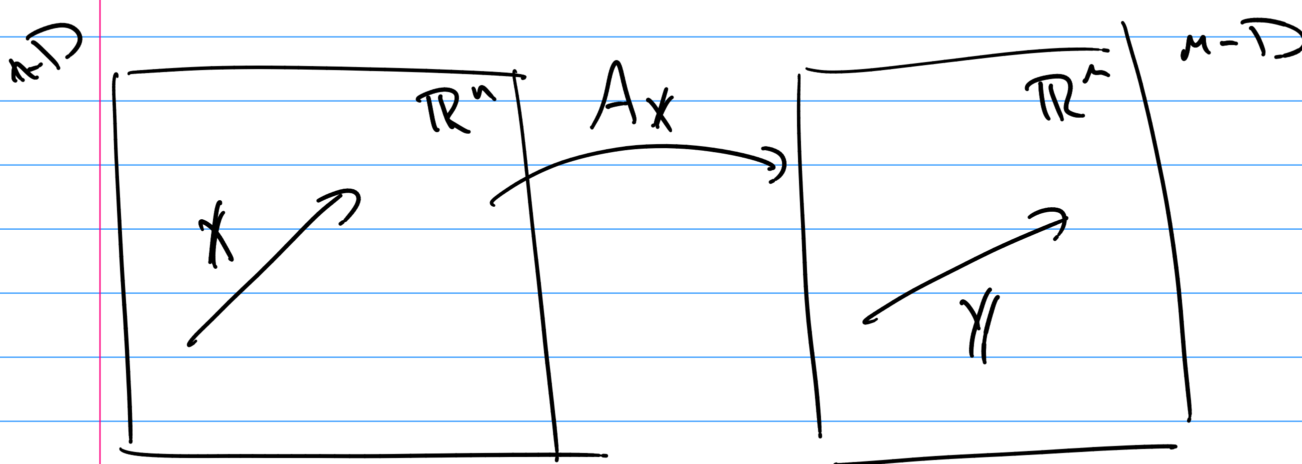
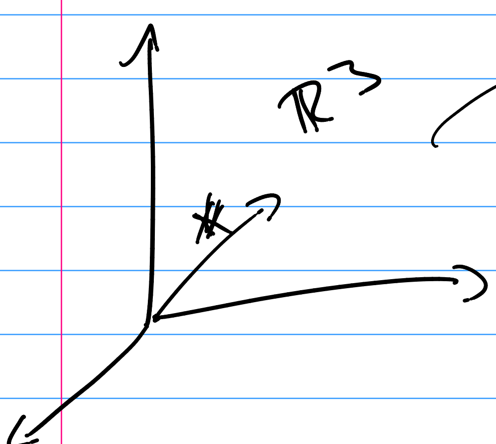
A Very Important Subspace

$A_{m \times n}$ consider $\underset{m \times n}{A} \underset{n \times 1}{\mathbf{x}} = \underset{m \times 1}{\mathbf{y}}$

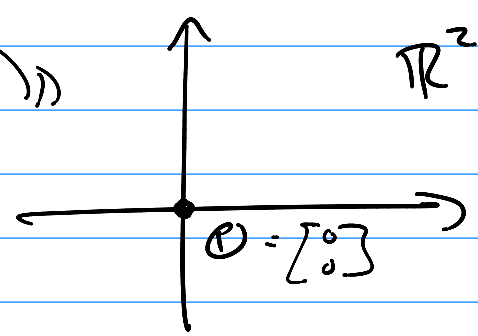
consider A to be a function



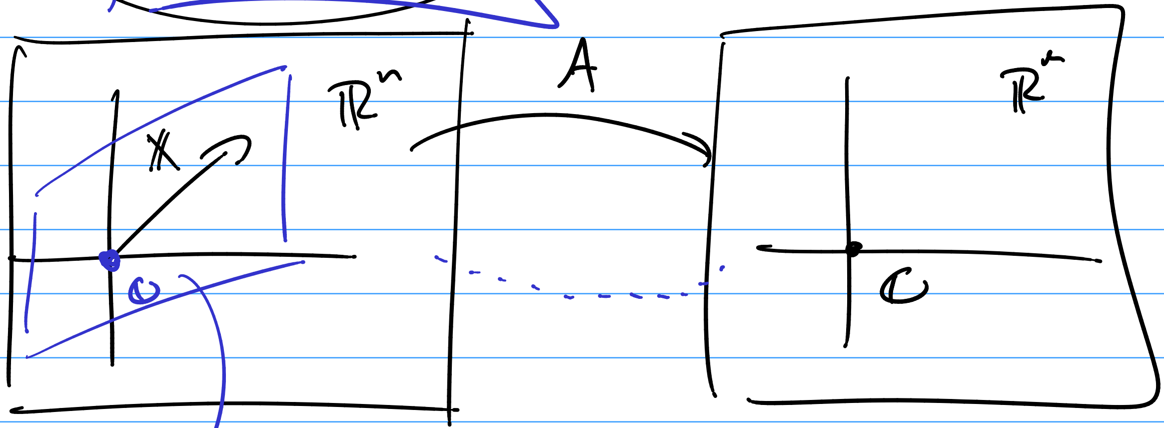


$\underset{2 \times 3}{A} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix}$

$A\mathbf{x}$



Note: $A\mathbf{x} = \mathbf{0}$ → find all $\mathbf{x}$ that solve this?



all non trivial (and $\mathbf{0}$) that $A\mathbf{x} = \mathbf{0}$

Its a subspace. Call it the null space of A.