Math 511

Q's 3.3 9d) e^{x}, e^{-x}, e^{2x} in $\boxed{C[0,1]}$ ← $\forall x \in [0,1]$

linear Ind $\quad \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_k v_k = 0$ ← Solve? $\alpha_i = ?$

$$\boxed{\alpha_1 e^{x} + \alpha_2 e^{-x} + \alpha_3 e^{2x} = 0} \rightarrow \text{3 unknowns but 1 eqn!}$$

$$\begin{array}{c} f \rightarrow \\ f' \rightarrow \\ f'' \rightarrow \end{array} \begin{bmatrix} e^{x} & e^{-x} & e^{2x} \\ e^{x} & -e^{-x} & 2e^{2x} \\ e^{x} & e^{-x} & 4e^{2x} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

use fact that $Ax = 0$ and $\det(A) \neq 0 \rightarrow$ only trivial soln

$$\begin{vmatrix} e^{x} & e^{-x} & e^{2x} \\ e^{x} & -e^{-x} & 2e^{2x} \\ e^{x} & e^{-x} & 4e^{2x} \end{vmatrix} = \begin{vmatrix} e^{x} & e^{-x} & e^{2x} \\ 0 & 2e^{-x} & -e^{2x} \\ 0 & 0 & 3e^{2x} \end{vmatrix} = (e^{x})(2e^{-x})(3e^{2x})$$

$$= 6e^{2x} \quad \text{on} \quad C([0,1])$$

← x's

if $x = 0$ $\quad |W| = 6 \neq 0$

so $\boxed{\text{linearly ind.}}$

$p(x) = a + bx + cx^2$ $\mathbb{R}^3$

3.4 14b

$\dim(P_3) = 3$

$p_1 = 0 + 1 \cdot x + 0 \cdot x^2$ $p_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$p_2 = -1 + 1 \cdot x + 0x^2$ $p_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$p_3 = 1 + 0 \cdot x + 1 \cdot x^2$ $p_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$p_4 = -1 + 0 \cdot x + 1 \cdot x^2$ $p_4 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$$P = \begin{bmatrix} 0 & -1 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

lead = keep this vector

free = dep. vector

$$\begin{bmatrix} 0 & -1 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow[\text{rref} - r_2]{\text{swap}} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$S = \operatorname{span}(\{p_1, p_2, p_3, p_4\}) = \operatorname{span}(\{p_1, p_2, p_3\})$$

$$\dim(S) = 3$$

3.5 #a

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

x 1 $2x-1$ $2x+1$

$P_2 = ax + b$

$P_2 = \begin{bmatrix} a \\ b \end{bmatrix}$

a) from $2x-1, 2x+1$ to $x, 1$

B_2 to B_1

$$[p]_{B_1} = B_1^{-1} B_2 [p]_{B_2}$$

$$[p]_{B_1} = \underbrace{\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} \right)}_{\text{transition matrix}} [p]_{B_2}$$

$$= \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$

So $\quad [P]_{B_1} = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} [P]_{B_2}$

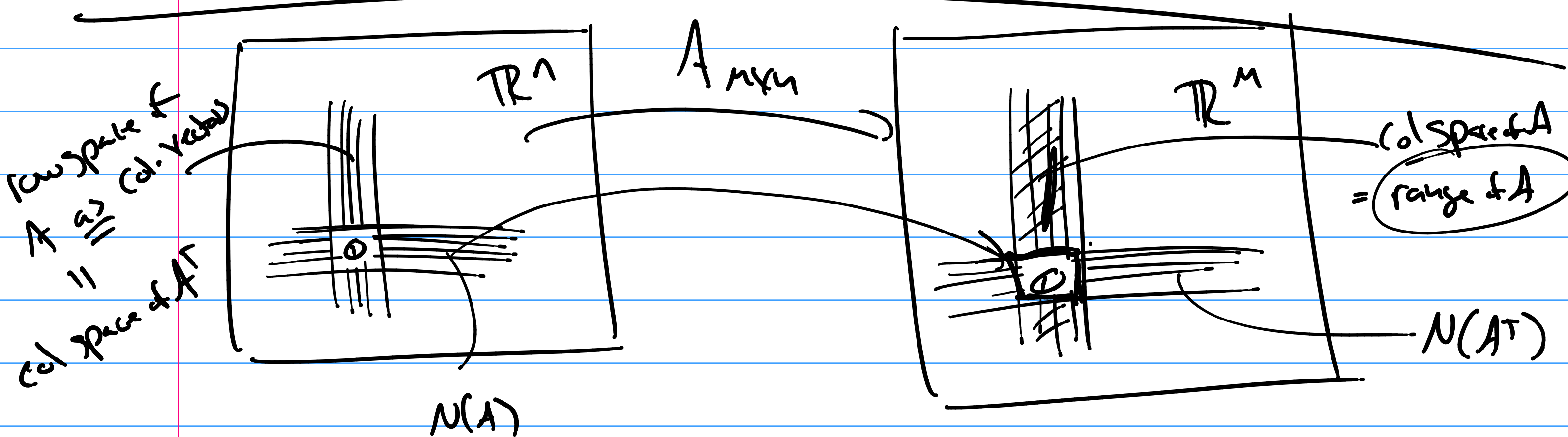
9b) $\quad [P] = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}^{-1} [P]_{B_1}$

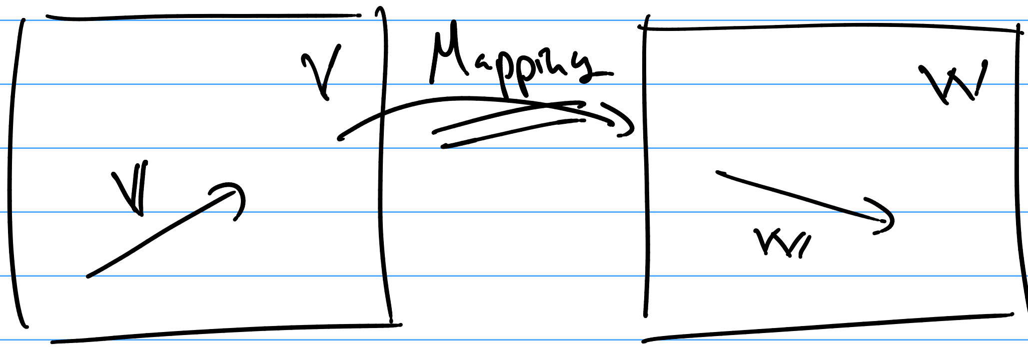
Find

$$\left[\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right]$$

$$\rightsquigarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & \text{9b)} \end{array}\right]$$

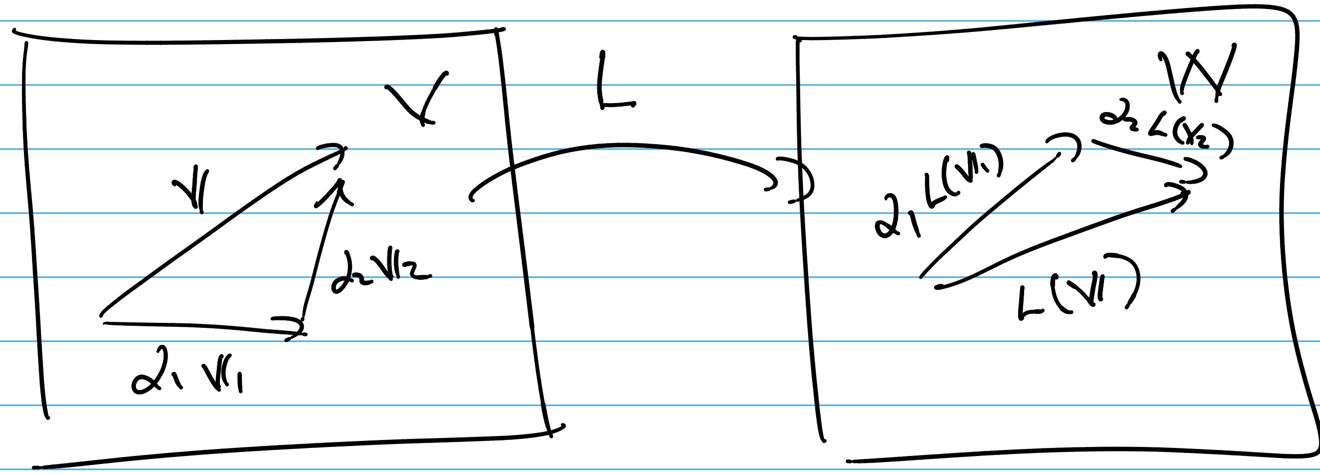




M is a mapping from V to W

if it takes $v \in V$ and maps it to $w \in W$

We will restrict our study of mappings to ones that ...



$\mathbb{v} = \alpha_1 \mathbb{v}_1 + \alpha_2 \mathbb{v}_2$ $\qquad$ $L(\mathbb{v}) = \alpha_1 L(\mathbb{v}_1) + \alpha_2 L(\mathbb{v}_2)$

ex $\quad A\mathbb{x} = \mathbb{y}$

Matrix mult. is distrib. and αA is assoc.

$$A(\alpha_1 \mathbb{v}_1 + \alpha_2 \mathbb{v}_2) = \alpha_1 A \mathbb{v}_1 + \alpha_2 A \mathbb{v}_2$$

Def: any mapping that preserves linear combinations is called a Linear transform.

two representations of this

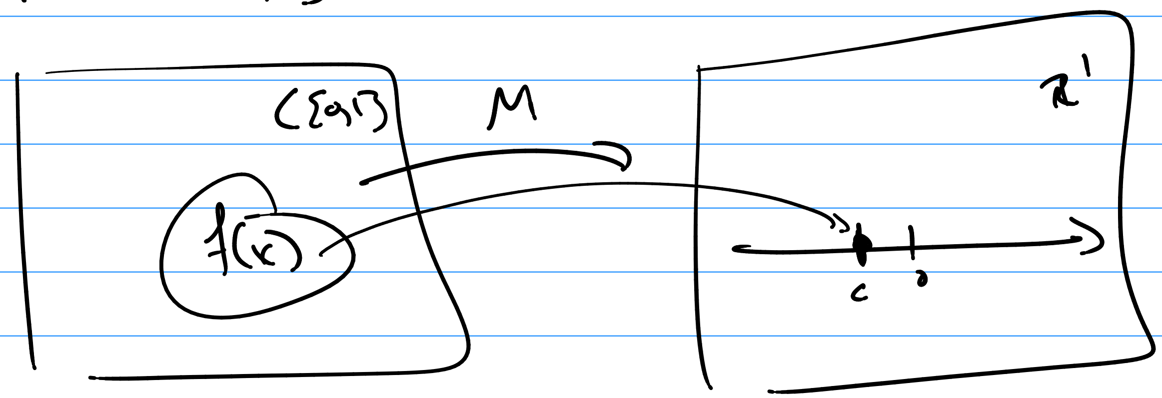
#1 $\quad L(\alpha_1 \mathbb{v}_1 + \alpha_2 \mathbb{v}_2) = \alpha_1 L(\mathbb{v}_1) + \alpha_2 L(\mathbb{v}_2)$

#2 $\quad$ (2 step version)

Step 1 $\quad L(\alpha \mathbb{v}) = \alpha L(\mathbb{v})$

Step 2 $\quad L(\mathbb{v}_1 + \mathbb{v}_2) = L(\mathbb{v}_1) + L(\mathbb{v}_2)$

ex $M : C[0,1] \to \mathbb{R}^1$



M maps $f(x)$ to its area over $[0,1]$

M is $\boxed{\int_0^1 f(x)\,dx}$

So check $L(\alpha_1 V_1 + \alpha_2 V_2) = \alpha_1 L(V_1) + \alpha_2 L(V_2)$

what is V? it is $f(x)$

V_1 let it be $f_1(x)$
V_2 let it be $f_2(x)$

what is L? it is $L(f(x)) = \int_0^1 f(x)\,dx$

So $L(\alpha_1 V_1 + \alpha_2 V_2)$ is $\boxed{\int_0^1 [\alpha_1 f_1(x) + \alpha_2 f_2(x)]\,dx}$

$\alpha_1 L(V_1) + \alpha_2 L(V_2)$ is $\boxed{\alpha_1 \int_0^1 f_1(x)\,dx + \alpha_2 \int_0^1 f_2(x)\,dx}$