Math 511

Solve $A\mathbb{x} = \mathbb{b}$ → 0 ans / 1 ans / ∞ ans] consistent

Q's (3.6 #6)

$A\mathbb{x} = \mathbb{b}$

① $\mathbb{b}$ is in col space of A

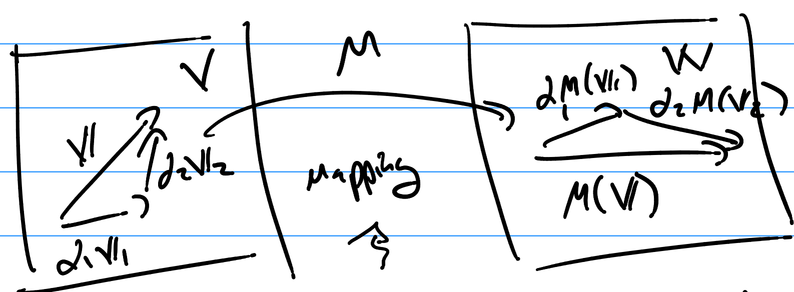
$\underline{\text{th}^m\ 3.6.2}$: $\mathbb{b} \in$ Col space of A ⟺ $A\mathbb{x} = \mathbb{b}$ is consistent ← has soln(s) $\underline{1 \text{ or } \infty}$

② A's cols $a_1, a_2, \ldots, a_n$ are linearly dep.

So our system $A\mathbb{x} = \mathbb{b}$ has free variables

→ $\underline{\infty \text{ solns}}$

4.1



if mapping preserves linear combos

→ call M a linear transformation

use $L: V \to W$ notation

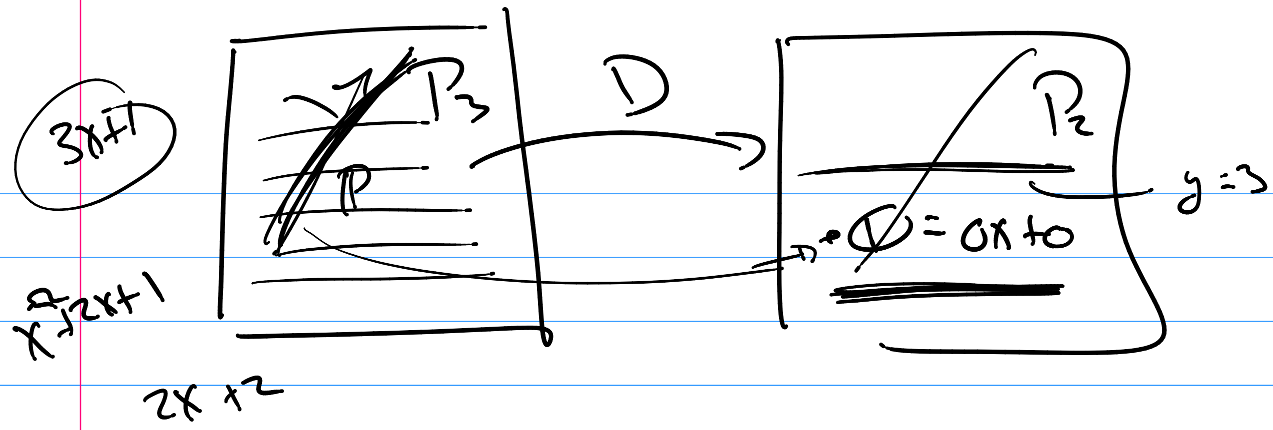
Ex: $\underline{\text{Deriv:}}$ $D: P_3 \to P_2$

$$D(3x^2 + 2x - 1) = 6x + 2$$

Linear trans? 2 step

#1 $D(\alpha(ax^2+bx+c)) = \alpha D(ax^2+bx+c)$ ok

#2 $D((ax^2+bx+c) + (dx^2+ex+f)) = D(ax^2+bx+c) + D(dx^2+ex+f)$ ok



Def Kernel of $L: V \to W$ is the set of all $v \in V$ such that $L(v) = 0_W$

ex from above $\ker(D) = \{ p \mid p = 0x^2 + 0x + c \}$
$= P_1$

Def ① S is a subspace of V

$L(S) = \{ w \in W \mid \text{for some } v \in S,\ L(v) = w \}$

L(S) is the image of S

② L(V) is the range of L

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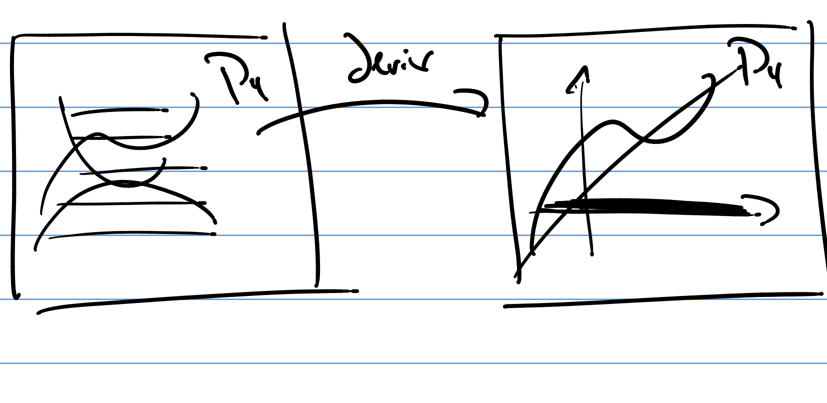
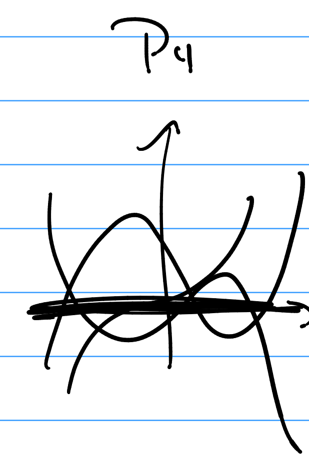
ex <u>Derivatives</u> $D: P_4 \to P_4$

$\ker(D) = P_1$

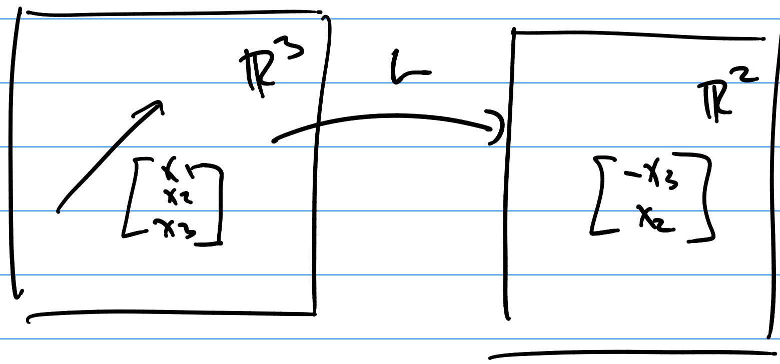
$L(D) = P_3$





Have: $\boxed{Ax = y \text{ is a linear transform}}$

$\boxed{4.2}$ if you have a linear transform..



is L a <u>linear trans?</u>

2 step check

#1 $L\left(\alpha \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) \stackrel{?}{=} \alpha L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)$

$L\left(\begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{bmatrix}\right) \stackrel{?}{=} \alpha \begin{bmatrix} -x_3 \\ x_2 \end{bmatrix}$

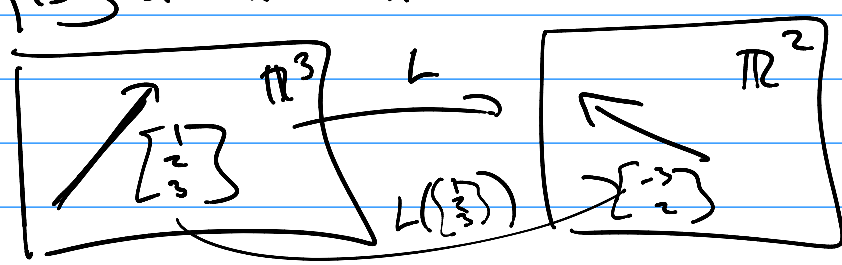
$\begin{bmatrix} -\alpha x_3 \\ \alpha x_2 \end{bmatrix} \stackrel{?}{=} \alpha \begin{bmatrix} -x_3 \\ x_2 \end{bmatrix}$ true!

#2 $L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix}\right) \stackrel{?}{=} L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) + L\left(\begin{bmatrix} d \\ e \\ f \end{bmatrix}\right)$

$L\left(\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix}\right) \stackrel{?}{=} \begin{bmatrix} -c \\ b \end{bmatrix} + \begin{bmatrix} -f \\ e \end{bmatrix}$

$\begin{bmatrix} -(c+f) \\ (b+e) \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} -c-f \\ b+e \end{bmatrix}$ true!

So L $\boxed{\text{is}}$ a linear trans!



Th^m for any $L: \mathbb{R}^n \to \mathbb{R}^m$ a linear transform

then there is a Matrix $A_{m \times n}$ such that

$$L(\mathbf{x}) = A\mathbf{x}$$

and in fact ... $[\mathbf{x}]_E = x_1 e_1 + x_2 e_2 + \dots + x_n e_n$

but $L([\mathbf{x}]_E) \underset{\text{b/c } L \text{ is a linear trans.}}{=} \boxed{x_1 L(e_1) + x_2 L(e_2) + \dots + x_n L(e_n)}$

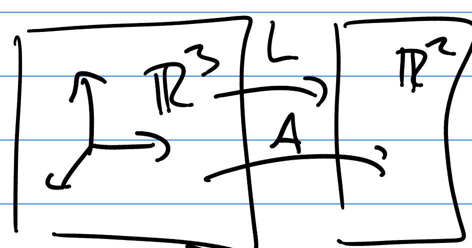
$$= \underline{\left[L(e_1)\ \ L(e_2)\ \cdots\ L(e_n)\right]} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= A\,[\mathbf{x}]_E$$

So $L([\mathbf{x}]_E) = A\,[\mathbf{x}]_E$

with $A = \left[L(e_1)\ \ L(e_2)\ \cdots\ L(e_n)\right]$

back to $\underline{L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_3 \\ x_2 \end{bmatrix}}$



find A? $A = \left[L\begin{pmatrix}1\\0\\0\end{pmatrix}, L\begin{pmatrix}0\\1\\0\end{pmatrix}, L\begin{pmatrix}0\\0\\1\end{pmatrix}\right]$

$$\boxed{A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}}$$

So standard matrix representation of L is

$$L([x]_E) = [y]_E$$

is

$$A[x]_E = [y]_E$$

$$A = \begin{bmatrix} L(e_1) & L(e_2) & \cdots & L(e_n) \end{bmatrix}$$

Coord notation in a basis is $[v]_{\beta}$

basis name

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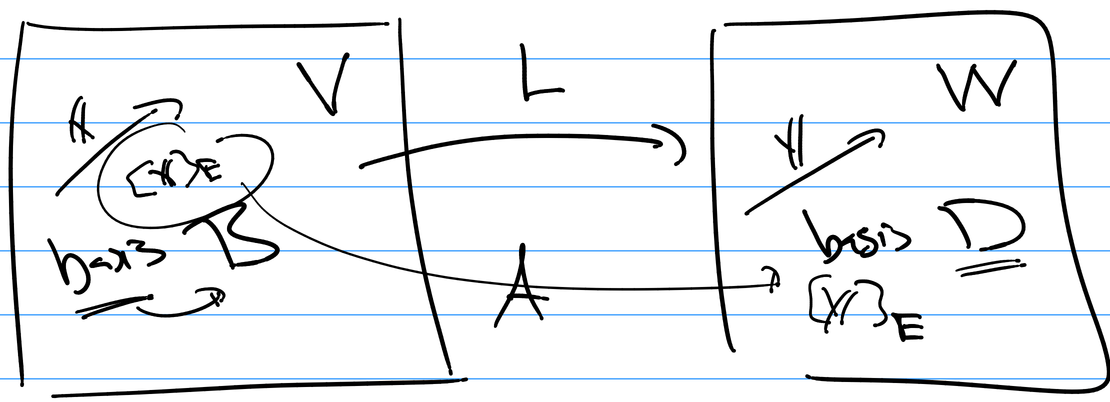
2 skills ① change basis

② A, standard rep of L.



$$L([x]_B) = [y]_D$$

Matrix?

$$\boxed{D^{-1}A\boxed{B}[x]_B} = [y]_D$$