# Math 511

**Qs** 3.6 #10 $\boxed{A_{m\times n}}$ rank$(A) = n$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & \_ & \cdots & a_{mn} \end{bmatrix}$$

$A\mathbf{x} = x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 + \cdots + x_n \mathbf{a}_n$

→ lin. indep. cols

→ uniq. linear combo

If $A\mathbf{c} = A\mathbf{d}$ $\exists$ does $\mathbf{c} = \mathbf{d}$?

$c_1 \mathbf{a}_1 + c_2 \mathbf{a}_2 + \cdots + c_n \mathbf{a}_n = d_1 \mathbf{a}_1 + d_2 \mathbf{a}_2 + \cdots + d_n \mathbf{a}_n$

b/c uniq. combos $\Rightarrow$ $\mathbf{c} = \mathbf{d}$.

(finish)

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$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\mathbf{a}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\mathbf{a}_3 = 1\mathbf{a}_1 - 1\mathbf{a}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$1\mathbf{a}_1 + 1\mathbf{a}_2 + 1\mathbf{a}_3 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$2\mathbf{a}_1 + 0\mathbf{a}_2 + 0\mathbf{a}_3 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

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4.2 #7 Identity operator? → index?

→ go back into text 4.2? 4.1?

example #8 $I(\mathbf{v}) = \mathbf{v}$

$\boxed{\mathbf{v} \to V} \xrightarrow{I} \boxed{\mathbf{v} \to V}$

i) $Y_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $Y_2 = \begin{bmatrix}1\\1\\0\end{bmatrix}$ $Y_3 = \begin{bmatrix}1\\0\\0\end{bmatrix} \to Y = \begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}$

ii) $T$ on $\mathbb{R}^3$

$T(P_1) = P_1$ $\rightsquigarrow$ $T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}_E\right) = \begin{bmatrix}1\\0\\0\end{bmatrix}_E$

$T(P_2) = P_2$ $\quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}_E\right) = \begin{bmatrix}0\\1\\0\end{bmatrix}_E$

$T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}_E\right) = \begin{bmatrix}0\\0\\1\end{bmatrix}_E$

in Y coord.?

change of Basis?

$$\boxed{\begin{aligned} B[V]_B &= [V]_E \\ B^{-1}[V]_E &= [V]_B \end{aligned}}$$

find!

So $\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}^{-1}\begin{bmatrix}1\\0\\0\end{bmatrix}_E = \begin{bmatrix}?\end{bmatrix}$ tech #1 a) find $\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}^{-1} \Rightarrow \left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\1&1&0&0&1&0\\1&0&0&0&0&1\end{array}\right]$

$\to$ use it

$\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}^{-1}\begin{bmatrix}0\\1\\0\end{bmatrix}_E = \begin{bmatrix}?\end{bmatrix}$ tech #2 $\left[\begin{array}{ccc|c}1&1&1&1\\1&1&0&0\\1&0&0&0\end{array}\right]$

(finish)

$\to \left[\begin{array}{ccc|c}1&0&0&\\0&1&0&\begin{bmatrix}1&1&1\\1&1&0\\1&0&0\end{bmatrix}^{-1}\begin{bmatrix}1\\0\\0\end{bmatrix}\\0&0&1&\end{array}\right]$

5) $A[x]_E = [x]_Y$

$\uparrow$

$Y^{-1}$

4.2 #1

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 - x_3 \\ 2x_2 - x_1 - x_3 \\ 2x_3 - x_1 - x_2 \end{bmatrix}$$ on $\mathbb{R}^3$

Standard basis?

Standard matrix $A = \left[ L\begin{pmatrix}1\\0\\0\end{pmatrix} \; L\begin{pmatrix}0\\1\\0\end{pmatrix} \; L\begin{pmatrix}0\\0\\1\end{pmatrix} \right]$ $\left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

a) $$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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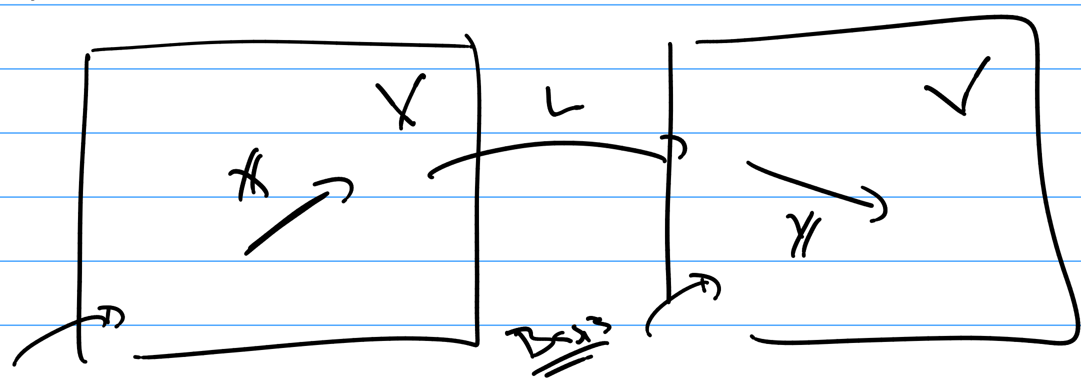
## $\boxed{4.2}$ Matrix representations of Linear operators on V

$\dim(V) = k$

① Standard matrix (use standard basis)

$$A = [\, L(e_1) \;\; L(e_2) \;\; .. \;\; L(e_k) \,]$$

② non-standard matrix (use non-standard basis, B)




Basis: E, B, D

Basis: E, B, D

$\{e_1, e_2 .. e_k\}$ $[b_1 \; b_2 .. b_k]$ $\{d_1 \; d_2 .. d_k\}$

$$L(x) = y$$

$A = [\, L(\vec{v}_1) \quad L(\vec{v}_2) \cdots L(\vec{v}_k) \,]$

Standard $\quad A[\vec{x}]_E = [\vec{y}]_E$

Basis B $\quad S[\vec{x}]_B = [\vec{y}]_B \qquad \underbrace{B^{-1} \underbrace{A \underbrace{B[\vec{x}]_B}_{[\vec{x}]_E}}_{[\vec{y}]_E}}_{S} = [\vec{y}]_B$

Basis D $\quad \underbrace{T}_{D^{-1}AD}[\vec{x}]_D = [\vec{y}]_D$

So $\boxed{L(\vec{x}) = \vec{y}}$ has 3 different matrix versions.

① $A[\vec{x}]_E = [\vec{y}]_E \qquad A = [\, L(\vec{v}_1) \quad L(\vec{v}_2) \cdots L(\vec{v}_k) \,]$

② $S = B^{-1}AB \qquad S[\vec{x}]_D = [\vec{y}]_B$

③ $T = D^{-1}AD \qquad T[\vec{x}]_D = [\vec{y}]_D$

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4.3 **Def**: given $A, B$ and a non-singular $S$ ($S^{-1}$ exists)

$$B = S^{-1} A S$$

call $A, B$ to be similar

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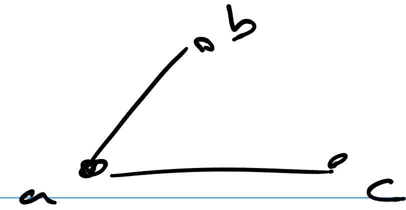
ex #1 So all the matrix reps of $L$ on $V$ are similar!

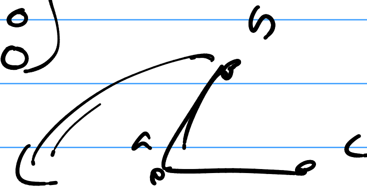
ex #2

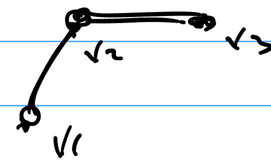
# Adjacency Matrices

graph




$$A_G = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$




## Isomorphism

same

move

$$A_{G_2} = E_{type1}^{-1} A_{G_1} E_{type 1}$$