Math 511 Exams + Final?

Before stay@home 3 exams + Final

Final was May 13^{th}

if grade was more than lowest exam → replace lowest exam

Now: exams 1 to 3 no ~~Final~~ ← drop lowest exam

Exam 3 open Friday 8^{th} AM

close Wed 13^{th} @ 11:30 pm

→ 6 probs in 4 hrs

Exam 3 6 probs

Ch 5 | 6

You do the work

software octave, Sage, Cocalc

Ideas

Ch 5 (4 probs)   Ch 6 (2 probs)

- Inner Products
- Orthogonal Subspaces (fundamental)
- Application (least squares data fitting)
- Orthonormal
- Gram-Schmidt

Problem 1 given 3 "vectors" in an inner product space.

ex
- $\mathbb{R}^n$; scalar prod
- $\mathbb{R}^{n\times n}$: weighted norm
- $C[a,b]$; $\langle f,g\rangle = \int_a^b fg\,dx$

→ can you find $\langle v_1, v_2\rangle$?

→ can use $\langle v_1, v_2\rangle$?

→ θ?   → length?

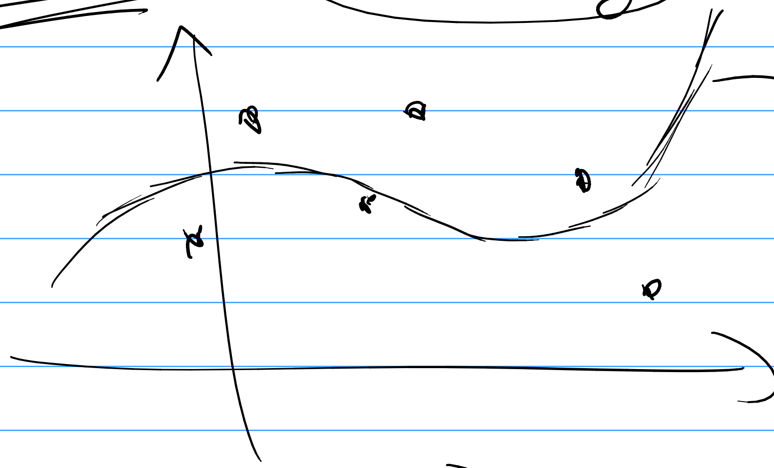
→ projection?

→ orthogonal?

Note: I may give you some vectors

Problem 2 data fitting

$$V^T V c = V^T y$$



— find "best" polynomial of a given degree

ex deg 3

$$p = \underbrace{a_1 + a_2 x + a_3 x^2 + a_4 x^3}_{4 \text{ terms}}$$

(Hint: Matrix solves?)

Problem 3 Orthonormal

Check
① orthogonal $v_i \perp v_j$
② length = 1 for all v_k

study $\mathbb{R}^n$, $\mathbb{R}^{m \times n}$ with innerproducts

show work

Problem 4 Gram-Schmidt $\mathbb{R}^4$ (by hand)

ex example 2 p. 268

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -1 \\ 4 \\ 4 \\ -1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 4 \\ -2 \\ 2 \\ 0 \end{bmatrix}$$

Similar

example 3 p. 270

→ generate u_1, u_2, u_3 (unit vectors)

$u_i \perp u_j$ (orthogonal)

→ orthonormal

Ch 6 (2 probs)

① Find eigenvalues / eigenvectors

② related rates diff eqn (6.2/6.3)

Ch6

(by hand)

Problem 1 Given $A \to \lambda_i$? (eigenvalue) x_i? (eigenvector)

Setup

Problem 2 $Y' = AY \quad Y(0) = Y_0$

Consider Example 1 (6.2) vs Example 7 (6.3) — Study!

$A \to$ eigenvalues λ, eigenvector x

$$\to A = \boxed{X}\boxed{D}\boxed{X^{-1}}$$

$X = [x_1 \ x_2 \cdots]$

$D = \begin{bmatrix} \lambda_1 & & 0 \\ 0 & & \lambda_k \end{bmatrix}$

6.3 **Soln** $\boxed{Y = X e^{tD} X^{-1} Y_0}$

example 7 p. 326

eigenvectors

$$Y = \begin{bmatrix} 4 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} e^{6t} & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 3 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

eigenvectors $e^{\text{eigenvalue } t}$ initial value

= Same as example 1 (6.2)

example 7 (6.3)