

Math 511

Row ops:

- ① Switch rows
- ② $\text{no row} = \text{new row}$
- ③ $\text{row}_i + \text{no row}_j = \text{new row}$

$$\left[\begin{array}{cccc|c} 0 & -1 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 6 \\ 2 & 4 & 1 & -2 & -1 \\ 3 & 1 & -2 & 2 & 3 \end{array} \right]$$

Solve:

① no soln
(inconsistent)

② soln (consistent)



$$\begin{aligned} x &= 1 \\ y &= 2 \\ z &= 3 \\ w &= 4 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right]$$

exactly 1

infinite

$$\left[\begin{array}{cccc|c} 0 & -1 & -1 & 1 & 0 \\ 2 & 4 & 1 & -2 & -1 \\ 3 & 1 & -2 & 2 & 3 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 0 & 6 \\ 1 & 1 & 1 & 1 & 6 \\ 2 & 4 & 1 & -2 & -1 \\ 3 & 1 & -2 & 2 & 3 \end{array} \right]$$

$$r_2 - r_1 = \text{New } r_2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 0 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -3 & -2 & -13 \\ 0 & -5 & -9 & 2 & -15 \end{array} \right]$$

$$r_1 - r_2 = \text{New } r_1$$

$$r_3 - 2r_1 = \text{New } r_3$$

$$r_4 - 3r_1 = \text{New } r_4$$

$$5r_2 + r_4 = \text{New } r_4$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 0 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & -3 & -2 & -13 \\ 0 & 0 & -3 & -3 & -15 \end{array} \right]$$

$$r_3 = \text{New } r_3$$

$$-\frac{1}{3}r_4 = \text{New } r_4$$

Swap r_3, r_4

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 0 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 5 \\ 0 & 0 & 3 & -2 & 13 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 0 & 6 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

all work till now
is called Gaussian elim.

Strict upper triangular

$$x + 2y + 2z + 0q = 6$$

$$y + z - q = 0$$

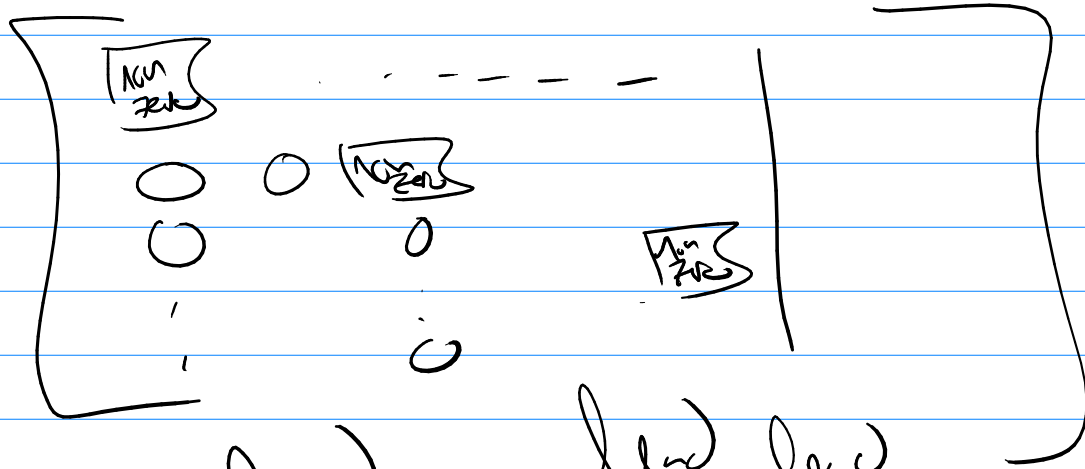
back
sub

$$z + q = 5$$

$$q = 2$$

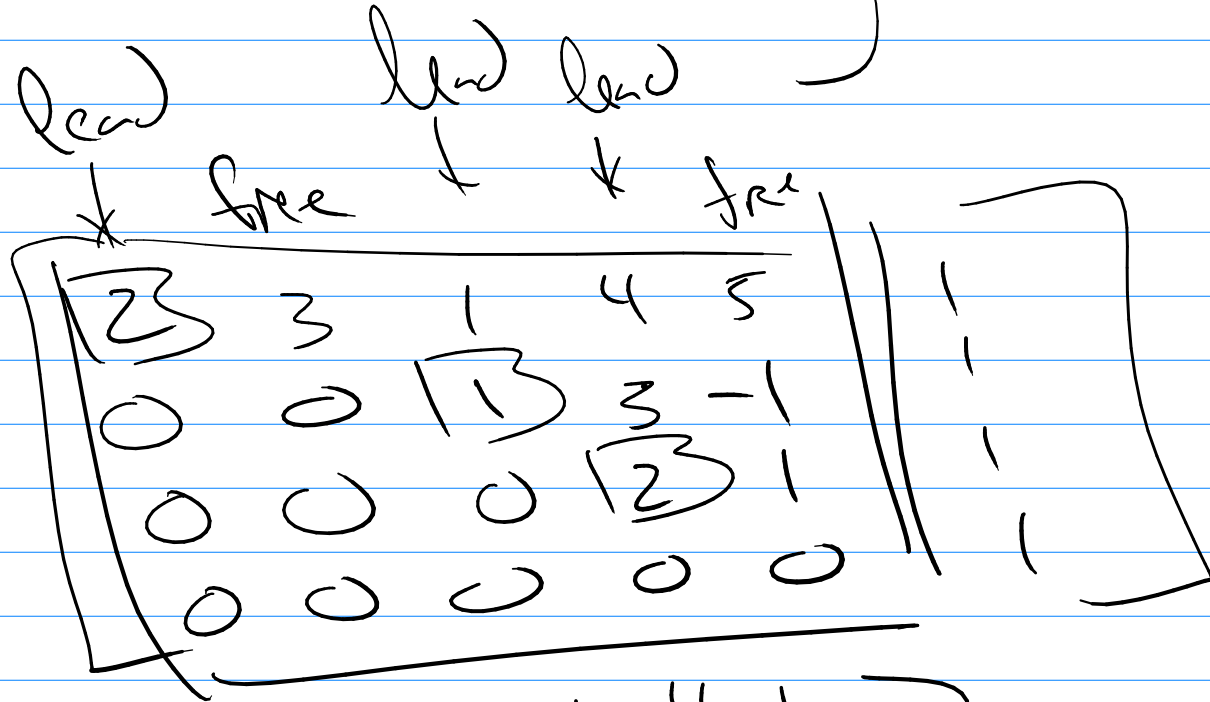
Note:

as you do row ops

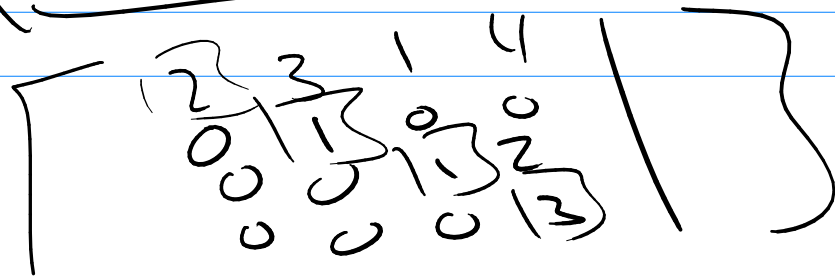


Start step look

row echelon



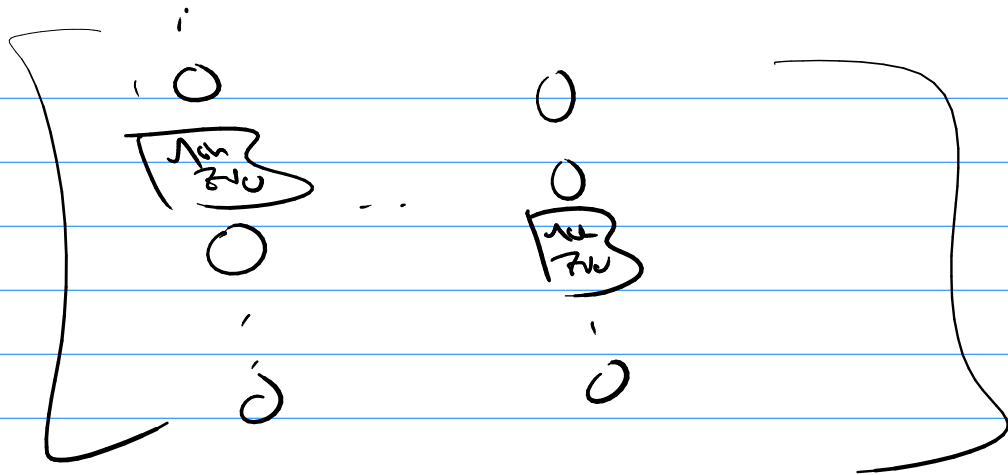
Start



reduced
row ech

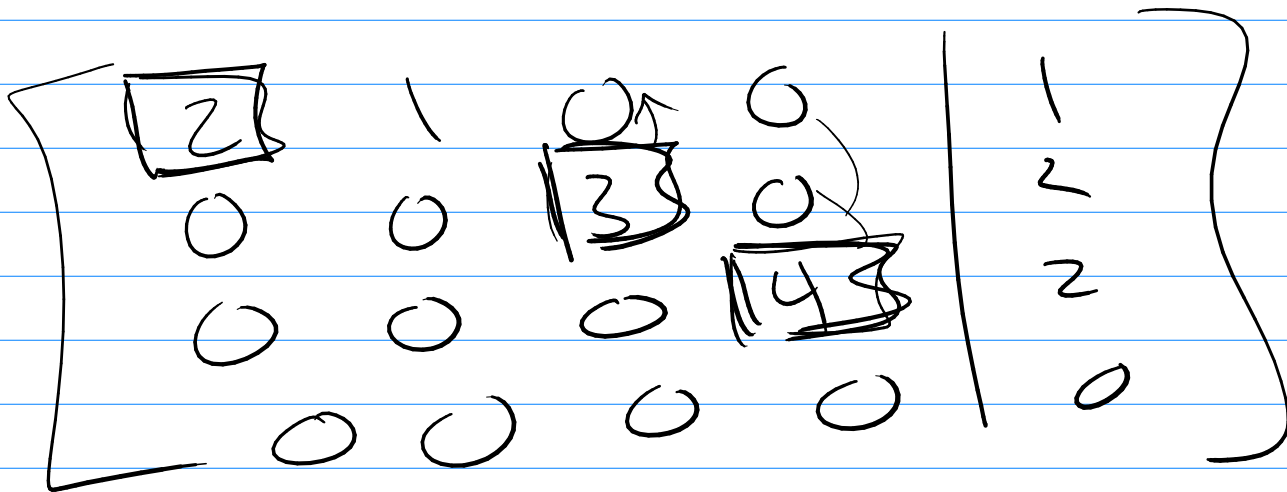
zeros above

and below
leads



(ex)

x



free as lead (Solve)

ex

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

No Soln

$$0 \cdot x + 0 \cdot y + 0 \cdot z + 0 \cdot w = 1$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

x_1 lead
 x_2 free
 x_3 lead
 x_4 free

so

$x_2 = \text{any thing}$

$x_4 = \text{any thing}$

$x_3 = 2$

$x_1 = 13$

$$x_3 = 3$$

$$x_1 + x_2 = 2$$

$$x_1 = 2 - \alpha$$

Sol

$$x_1 = 2 - \alpha$$

$$x_2 = \alpha$$

$$x_3 = 3$$

$$x_1 = \beta$$

↳

$$x_1 = 2 - \alpha + 0\beta$$

$$x_2 = 0 + \alpha + 0\beta$$

$$x_3 = 3 + 0\alpha + 0\beta$$

$$x_1 = 0 + 0\alpha + 1\beta$$
