Math 511

Q5

$$\begin{aligned} x_1 + 3x_2 + x_3 + x_4 &= 3 \\ 2x_1 - 2x_2 + x_3 + 2x_4 &= 8 \\ 3x_1 + x_2 + 2x_3 - x_4 &= -1 \end{aligned}$$

Solve

1a) No solution

2a) exactly 1

2b) infinite solutions

$$\left[\begin{array}{cccc|c} \boxed{1} & 3 & 1 & 1 & 3 \\ 2 & -2 & 1 & 2 & 8 \\ 3 & 1 & 2 & -1 & -1 \end{array}\right]$$

3×4

eqn's < vars

Underdetermined

row ech

$N r_2 = -2 r_1 + r_2$

$N r_3 = -3 r_1 + r_3$

$$\left[\begin{array}{cccc|c} \boxed{1} & 3 & 1 & 1 & 3 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & -8 & -1 & -4 & -10 \end{array}\right]$$

$N r_3 = -1 r_2 + r_3$

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & 0 & 0 & -4 & -12 \end{array}\right]$$

row ech.

free $= x_3$

$$\left[\begin{array}{cccc|c} \boxed{1} & 3 & 1 & 1 & 3 \\ 0 & \boxed{8} & 1 & 0 & -2 \\ 0 & 0 & 0 & \boxed{1} & 3 \end{array}\right]$$

$x_3 = \alpha$ $\quad x_1 + 3\boxed{x_2} + 2\alpha + x_4 = 3$

$8x_2 + \alpha = -2 \rightarrow x_2 = \dfrac{-2-\alpha}{8}$

$x_4 = \boxed{3}$

gaussian elim

reduced row

Make zeros above leads

$$\left[\begin{array}{cccc|c} \boxed{1} & 3 & 1 & 1 & 3 \\ 0 & \boxed{8} & 1 & 0 & -2 \\ 0 & 0 & 0 & \boxed{1} & 3 \end{array}\right] \quad Nr_1 = r_1 - r_3$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 0 \\ 0 & \boxed{8} & 1 & 0 & -2 \\ 0 & 0 & 0 & \boxed{1} & 3 \end{array}\right] \quad Nr_1 = 3r_2 - 8r_1$$

$\alpha = x_3$

$$\left[\begin{array}{cccc|c} -8 & 0 & -5 & 0 & -6 \\ 0 & 8 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 5/8 & 0 & 3/4 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$x_1 = 3/4 - 5/8\, \alpha$

$x_2 = -7/4 - 1/8\, \alpha$

$x_3 = \alpha$

$x_4 = 3$

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ex

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 3 \\ 0 & \boxed{1} & 2 & 4 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

row says $0 = 5$

No Soln

free $\quad x_3 = \alpha$

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 3 \\ 0 & \boxed{1} & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

says problem is really 2×3

5×3

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 3 & 1 \\ 0 & \boxed{2} & 4 & 2 \\ 0 & 0 & \boxed{3} & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

really a 3×3

no free

$\to$ exactly one soln

$$\left[\begin{array}{cc|c} \boxed{1} & 1 & -1 \\ 1 & -\cancel{3}\,4 & \cancel{3}\,3 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|c} \boxed{1} & 1 & -1 \\ 0 & \boxed{-7} & 7 \end{array}\right]$$

row ech

$Nr_1 = 7r_1 + r_2$

$$\begin{array}{c} \frac{1}{7} \\ -\frac{1}{7} \end{array} \left[\begin{array}{cc|c} \boxed{7} & 0 & 0 \\ 0 & \boxed{-7} & 7 \end{array}\right] \longleftrightarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -1 \end{array}\right]$$

$x_1 = 0$

$x_2 = -1$

$Nr_1 = r_1 - r_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & -1 & 0 \end{array}\right]$$

Note:

Homogeneous Systems

all eqn's are $= 0$

always have all **zeros** soln.

trivial soln.

$$x_1 + x_2 + x_3 = 0$$
$$x_1 - x_2 - x_3 = 0$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \end{array}\right]$$

x_3 is free $\quad x_3 = \alpha$

x_1 is lead $\quad x_2$ is a lead

$x_3 = \alpha$

$x_2 + x_3 = 0$

$x_2 + \alpha = 0$

$x_2 = -\alpha$

$x_1 + x_2 + x_3 = 0$

$x_1 - \alpha + \alpha = 0$

$x_1 = 0$

$x_1 = 0$

$x_2 = -\alpha$

$x_3 = \alpha$

$(0, -\alpha, \alpha)$

$$X = \begin{bmatrix} 0 \\ -\alpha \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$4 \times 4 \rightsquigarrow \left[\begin{array}{cccc|c} \boxed{1} & 2 & 3 & 4 & 5 \\ 0 & 0 & \boxed{1} & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

x_1, x_3 are leads

x_2, x_4 are free $\longrightarrow$ $x_2 = \alpha$

$x_4 = \beta$