Math 511

3 cr ~> $3 \times 3 \approx 9$ hrs / wk

$3 \times 6 \approx 18$ hrs / wk ~ 2.5 / 3 hrs / day

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already 1.4 watched / read

6/5 now ask questions

finish 1.4 read / watch 1.5

Matrix Algebra

toys: Matrix / vectors $\vec{v}$, $\vec{v}$

ops: $+$, $\cdot$, A^T, αA

$$A - B = A + (-1)B$$

ex Col. algebra

$$3x + 4 = 4$$
$$-4 \quad -4$$
$$3x = 0$$
$$3 \quad 3$$
$$x = 0$$

Expression: $3A + B = C$

$A = ?$

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

ans $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Undo | do — oper

Inverse operation — know identity = do nothing

$$A + B = C$$

Matrix add

Identity $= 0$

$$= \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}$$

$$A + B = C$$
$$\quad -B \qquad -B$$

Additive Inverse: of A

$(-1)A$

$$A + B + (-1)B = C + (-1)B$$

$$A + 0 = C + (-1)B$$

$$A + (-1)A = 0$$

$$A = C + (-1)B = C - B$$

$$BB^{-1} = I$$
$$B^{-1}B = I$$

$$AB = C$$

inv(B)

$$AB B^{-1} = C B^{-1}$$

$$A I = C B^{-1}$$

$$\boxed{A = CB^{-1}}$$

$$AX + (-1)D = E$$

$$AX = E + D$$

$$A \underbrace{X X^{-1}}_{I} = (E + D) X^{-1}$$

$$A = (E + D) X^{-1}$$

Special Matrix: (1) $O = \begin{bmatrix} \text{all zeros} \end{bmatrix}$ Matrix additive identity

Matrix mult. dominator.

$$A + O = A$$

$$A \cdot O = O$$

(2) $I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$ ones on diag, zeros everywhere else

Matrix mult. identity

$$\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$\underset{3\times 4}{A}\, I_4 = A \qquad I_3 \underset{3\times 4}{A} = A$$

Book & Jay

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & 1 & \beta & 0 \end{array}\right]$$

homogeneous → always consistent

↓

so only trivial soln $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

or

infinite soln

? free variables?

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 3 & 1+\beta & 0 \end{array}\right]$$

$\beta \neq 2$ → only trivial soln

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & \boxed{\beta - 2} & 0 \end{array}\right]$$

lead means non-zero

$\beta = 2$

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

free

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a_{ij}

↑ row col

$$\begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} 1\;2\;3 \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} 0 & & \\ 0 & & \\ 0 & & \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

$2 \times 3 \quad 3 \times 7 \quad 2 \times 7$

1:3 2×

$$\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} & \\ & \\ & \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

$2 \times 3 \quad 3 \times 2 \quad 2 \times 2$

$$[\;][\;] = [\;]$$

$$a \times b \quad b \times c \rightarrow a \times c$$

$$[\;][\;] = [\;]$$

ex $\quad 3 \times 3 \quad 3 \times 3 \rightarrow 3 \times 3$

$$= [\;][\;] = [10]$$

$$2 \times 4 \neq 8 \times 3$$