

Math 511

1.4 #12 We can't find A^{-1} . We can show A^{-1} by $AA^{-1} = A^{-1}A = I$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{d} a_{22} & -\frac{1}{d} a_{12} \\ -\frac{1}{d} a_{21} & \frac{1}{d} a_{11} \end{bmatrix} \quad \text{with } d = (a_{11} a_{22} - a_{12} a_{21})$$

$$\rightarrow \frac{1}{d} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} a_{22} - a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \frac{1}{d} \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix} = I$$

1.4 #19 Give $A^2 = \underline{AA} = 0$ is all zero

show $(I + A) = (I - A)^{-1}$ $\begin{matrix} \nwarrow \\ \nearrow \end{matrix}$ $\begin{matrix} Q \cdot I = Q \\ I \cdot Q = Q \end{matrix}$

$$\textcircled{\#1} \quad \boxed{(I + A)(I - A)} = I \cdot I - \boxed{IA} + \boxed{AI} - A \cdot A$$

$$\begin{aligned} &= I - \boxed{A} + A - 0 \\ &= I + 0 - 0 = \boxed{I} \end{aligned}$$

$\textcircled{\#2} \quad (I - A)(I + A) = I$

goal: $x=1, y=2, z=1$

$$\begin{cases} 2x + y - z = 3 \\ x - y + 2z = 1 \\ 3x + y + z = 6 \end{cases}$$

or swap r_2

$$\begin{cases} x - y + 2z = 1 \\ 2x + y - z = 3 \\ 3x + y + z = 6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 3 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & 1 & 6 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 2 & 1 & -1 & 3 \\ 3 & 1 & 1 & 6 \end{array} \right]$$

$$A X = I b$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$$

$$\begin{cases} x - y + z = 1 \\ 3y - 5z = 1 \\ 4y - 5z = 3 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & -1 & 2 & 1 \\ 0 & 3 & -5 & 1 \\ 0 & 4 & -5 & 3 \end{array} \right]$$

$$\begin{aligned} r_2 + (-2)r_1 &= 11r_2 \\ r_3 + (-2)r_1 &= 11r_3 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array} \right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array} \right] \left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array} \right] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}$$

$$E_3 E_2 E_1 A X = E_3 E_2 E_1 I b$$

E_{type1} E_{type2}

$$\begin{cases} x = 1 \\ y = 2 \\ z = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array} \right]$$

$$\begin{aligned} (E_k \dots E_1) A X &= E_k \dots E_1 I b \\ I X &= A^{-1} I b \end{aligned}$$

Why???

$$\begin{bmatrix} I_k & -F_2 F_1 \end{bmatrix} A x = \begin{bmatrix} F_1 & -F_2 F_1 B \end{bmatrix}$$

$$\downarrow \boxed{I} x = \begin{bmatrix} 1 \\ \vdots \end{bmatrix} x$$

⊗ Augmented Matrix = faster learner

2 reasons ① Use Augmented Matrices to find A^{-1} or multiply by A^{-1}

or

⊗ $[A | C]$

row ops

$$[I | A^{-1} C]$$

$$AB = C$$

$$\begin{bmatrix} I_k & -F_2 \end{bmatrix} A B = \begin{bmatrix} F_1 & -F_2 B \end{bmatrix} C$$

$$I B = A^{-1} C$$

$$AX = C$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$X = A^{-1} C$$

$$\rightarrow [A | I]$$

$$\downarrow$$

$$[I | A^{-1}]$$

$$[A | C]$$

$$\downarrow$$

$$[I | A^{-1} C]$$

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A X = C$$

$$X = \underline{\underline{A^{-1}C}}$$

①
Find A^{-1}

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 3 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1/3 & 1/3 \end{array} \right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1/3 & -2/3 \\ 0 & 1 & 1/3 & 1/3 \end{array} \right]$$

A^{-1}

$A \quad C$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array} \right]$$

② Find $A^{-1}C$ in one step

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 1 \\ 0 & 3 & 1 & 2 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 2 & 1 & 1 \\ 0 & 1 & 1/3 & 2/3 \end{array} \right] = \left[\begin{array}{cc|cc} 1 & 0 & 1/3 & -2/3 \\ 0 & 1 & 1/3 & 2/3 \end{array} \right]$$

I

$A^{-1}C$

②

Factoring

$$[A | \underbrace{\quad}_{\text{ignore}}]$$

Gauss

$$[U | \quad]$$

$$[E_k \dots E_2 E_1 A] x = b$$

$$\text{So } U = E_k \dots E_2 E_1 A$$

$$[E_1^{-1} E_2^{-1} \dots E_k^{-1}] U = A$$

mult. of matrices = A (factoring!)

Note If you only use type 3 ops each E_i is lower triangular and E_i^{-1} is lower triangular and $E_k^{-1} \dots E_1^{-1}$ is lower triangular

$$\text{So } A = \underbrace{[E_k^{-1} \dots E_1^{-1}]}_{L} \underbrace{U}_{U}$$

$$L = 2 \times 3$$

$$A = (U)$$