Math 511

$\boxed{Q\text{'s}}$

$A^{-1}A = I$

$\underbrace{[E_4E_3E_2E_1]}_{\text{mult}}A = I$

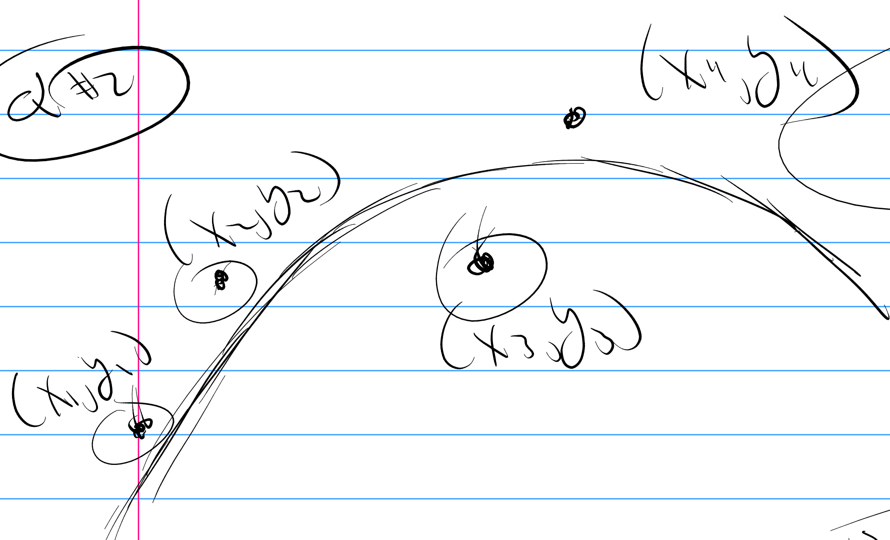
$A^{-1}\cdot A = I$

$\cancel{E_4^{-1}E_4}E_3E_2E_1A = E_4^{-1}I$

$\cancel{E_3^{-1}E_3}E_2E_1A = E_3^{-1}E_4^{-1}I$

$A = E_1^{-1}E_2^{-1}E_3^{-1}E_4^{-1}$

Q#2



$y = a + bx + cx^2$

$$V = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$$

$\det(V)$

2.1

2.2

$$\det\begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{pmatrix}$$

cofact

$$= 1\begin{vmatrix} x_2 & x_2^2 \\ x_3 & x_3^2 \end{vmatrix} - 1\begin{vmatrix} x_1 & x_1^2 \\ x_3 & x_3^2 \end{vmatrix} + 1\begin{vmatrix} x_1 & x_1^2 \\ x_2 & x_2^2 \end{vmatrix} =$$

$$= (x_2x_3^2 - x_3x_2^2) - (x_1x_3^2 - x_3x_1^2) + (x_1x_2^2 - x_2x_1^2)$$

$$=$$

type 3 Row equiv

$$\det\begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{pmatrix} = \det\begin{pmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2-x_1 & x_2^2-x_1^2 \\ 0 & x_3-x_1 & x_3^2-x_1^2 \end{pmatrix}$$

$$= (x_2-x_1)\underbrace{(x_3^2-x_1^2)}_{(x_3-x_1)(x_3+x_1)} - (x_3-x_1)\underbrace{(x_2^2-x_1^2)}_{(x_2-x_1)(x_2+x_1)}$$

$$= (x_2-x_1)(x_3-x_1)\left[(x_3+\cancel{x_1}) - (x_2+\cancel{x_1})\right]$$

$$\det(V) = (x_2 - x_1)(x_3 - x_1)(x_3 - x_2) \neq 0$$

Non-singular?

Singular?

Facts:

A is square

$\det(A) = 0$	$\det(A) \neq 0$
A^{-1} does not exist	A^{-1} does exist
A is called singular	A is called non-singular
A is not invertible	A is invertible
$Ax = 0$ has non-trivial (and infinite) solutions	$Ax = 0$ has only trivial
Ax has free vars	Ax has no free vars

$$[A \mid I]^+ = \begin{bmatrix} A^+ \\ \hline I^T \end{bmatrix}$$

2×1

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right]^T = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ \hline 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}^T \\ \hline \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T \end{bmatrix}$$

examples

1.6 not on exam

$$I^+ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^+ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$[A \mid I]^+ = \begin{bmatrix} A^+ \\ \hline I^+ \end{bmatrix}$$

2×1

$$\to \begin{bmatrix} A^+ \\ \hline I^+ \end{bmatrix} [C \mid D] = \left[\begin{array}{c|c} A^+C & A^+D \\ \hline C & D \end{array}\right]$$

$\boxed{\det(A)}$ Know ch2 2.1 / 2.2

① Do the $\det(A)$

$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

i) $\boxed{\text{Cofactor}}$

$$\det\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 1 & 2 & 0 \end{pmatrix} = -0\cancel{\begin{vmatrix} 2 & 3 \\ 2 & 0 \end{vmatrix}} + (-1)\begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} - \cancel{0\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}}$$

$= A$

$$= -1\begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} = -1(-3) = \boxed{3}$$

$$A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{x} = A^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2) $\boxed{\text{elimination}}$

$$\det\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 1 & 2 & 0 \end{pmatrix} \overset{\text{type 3 row equiv}}{=} \det\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{pmatrix} = (1)(-1)(-3)$$

$$= \boxed{3}$$

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Exam

0) Start time

Sign name :

w/ ID

(6) 1 2 3 6 4 5 6

1) 4x4 Solve use No Matrices

do

3x3

$$\begin{cases} x_1 + x_2 + x_3 = 2 \\ x_1 - 2x_2 - 2x_3 = -1 \\ 2x_1 + x_2 - 3x_3 = 7 \end{cases} \longrightarrow x_1 = 2 - x_2 - x_3$$

2) Same as 1) use aug. Matrices guess

(3) traffic (look @ book)

(4)
(5) > opps

(a) $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T + \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} + \left(\begin{bmatrix} x & 2 \\ 3 & 5 \end{bmatrix}\right)$$

(a) $$\underset{3\times 1}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^{T}} \underset{1\times 3}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} \; \end{bmatrix}}$$

(a) $$3\begin{bmatrix} x & 2 \\ -y & x \end{bmatrix} + \underset{2\times 1}{\begin{bmatrix} x \\ x \end{bmatrix}} \underset{1\times 4}{\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}} =$$

⑥ ex

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 0 \end{bmatrix} = \left[\begin{array}{c|c} & \\ \hline & \end{array}\right]$$

$2\times3 \quad 3\times2 \quad 2\times2$

⑥ ex

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix} = ?$$ nope

$1\times3 \quad 2\times3$

⑥ Many

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Find X

$$XC + AB = 3B$$

(7) LU (see last session)

(8) find A^{-1}

(9) a) $\det(A)$ by cof.

b) $\det(A)$ by elimination

c) interpret $\quad Ax = 0$

(10) $\det(A)$ → variables like today's Q#2

→ interpret

(11) $\det(A)$ vs $Ax = b$ vs $Ax = 0$

$0 = (x_1 a_{11} + x_2 a_{12} + \dots + x_n a_{1n}$

$$\det\begin{pmatrix} 2 & -1 & 0 \\ 1 & 3 & 1 \\ 3 & -1 & 2 \end{pmatrix} \overset{\text{type 1 equiv}}{=} -\det\begin{pmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 3 & -1 & 2 \end{pmatrix}$$

type 3

$r_2 + (-2)r_1 \to r_2$

$$\overset{\text{type 3 equiv}}{=} -\det\begin{pmatrix} 1 & 3 & 1 \\ 0 & -7 & -2 \\ 3 & -1 & 2 \end{pmatrix}$$