Math 511

Change of basis (pattern of what is going on?!)

$$[v]_E = c_1 b_1 + c_2 b_2 + c_3 b_3 + \dots + c_k b_k$$

(1) $[v]_E = B \begin{bmatrix} c_1 \\ \vdots \\ c_k \end{bmatrix}_B$ Know (2) $\begin{bmatrix} c_1 \\ \vdots \\ c_k \end{bmatrix}_B = B^{-1} [\;]_E$

3.5 #6

$$v_1 = \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \rightarrow V = \begin{bmatrix} 4 & 0 & 0 \\ 6 & 1 & 1 \\ 7 & 1 & 2 \end{bmatrix}$$

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad u_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad u_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \rightarrow U = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

a) transition matrix from v_i to u_i

$$[\text{coord}]_u = S\,[\text{coord}]_v \qquad [x]_u = U^{-1}V\,[x]_v$$

$$[x]_u = U^{-1}\underbrace{V[x]_v}_{[x]_E}$$

So $\boxed{S = U^{-1}V}$

to go from u to v?

transition from u to v

$$[x]_v = \overbrace{V^{-1}U}\,[x]_u$$

$(*)$ $p = cx + d$, $p = \begin{bmatrix} d \\ c \end{bmatrix}$ $B_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $B_2 = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

3.5 #4 $B_1 = [x, 1]$ $B_2 = [2x-1, 2x+1]$ in P_2

So $B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B_2 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$

$\mathbb{R}^2$: $p = ax + b$ or $p = \begin{bmatrix} a \\ b \end{bmatrix}$ (x, const)

(a) from B_2 to B_1 $\boxed{[p]_{B_1} = B_1^{-1} B_2 [p]_{B_2}}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = S}$

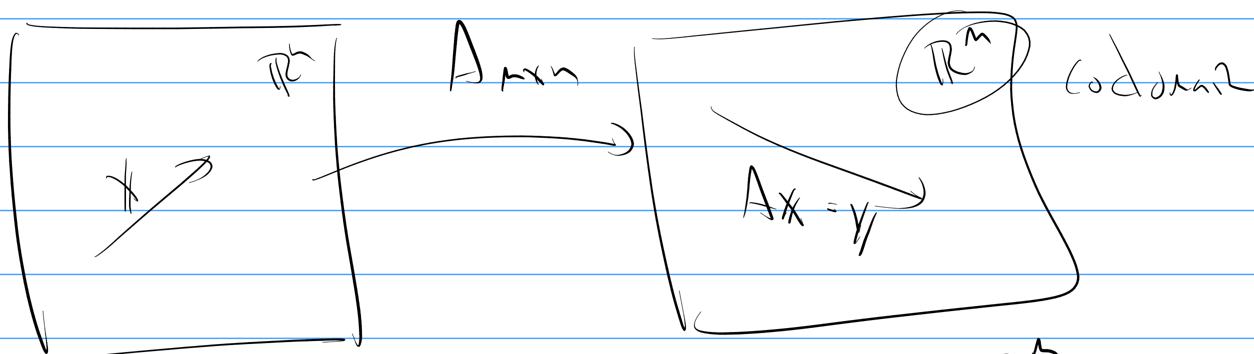
(b) from B_1 to B_2

$[p]_{B_2} = B_2^{-1} B_1 [p]_{B_1}$

$\boxed{\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$

$$S = B_1^{-1} B_2$$

Note: $B_2^{-1} B_1 = S^{-1}$



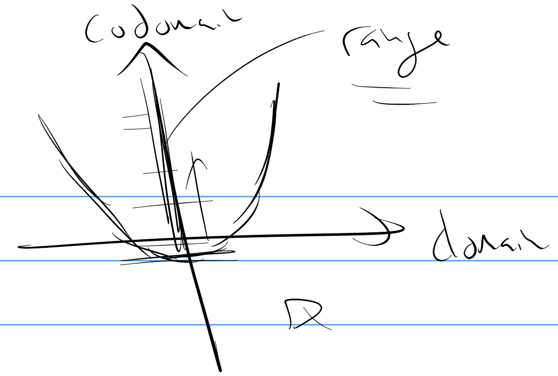
$x \in \mathbb{R}^n$ $y \in \mathbb{R}^m$

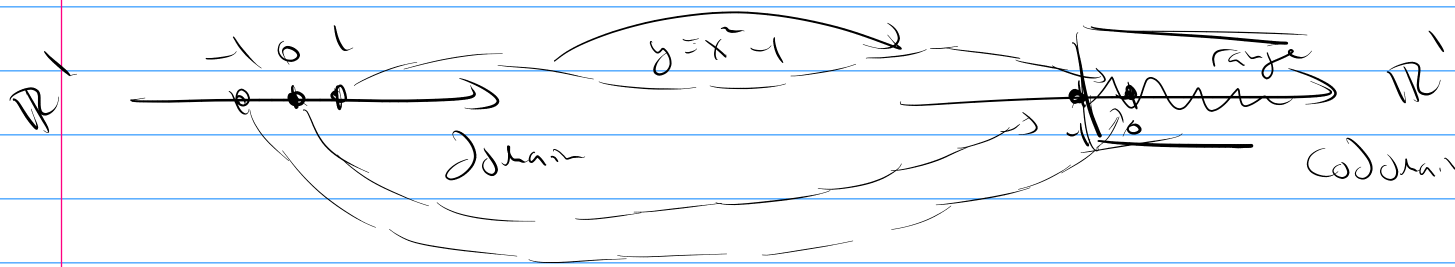
$$\underset{2\times 4}{\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 2 & 3 \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 2 \\ 6 \end{bmatrix}}$$

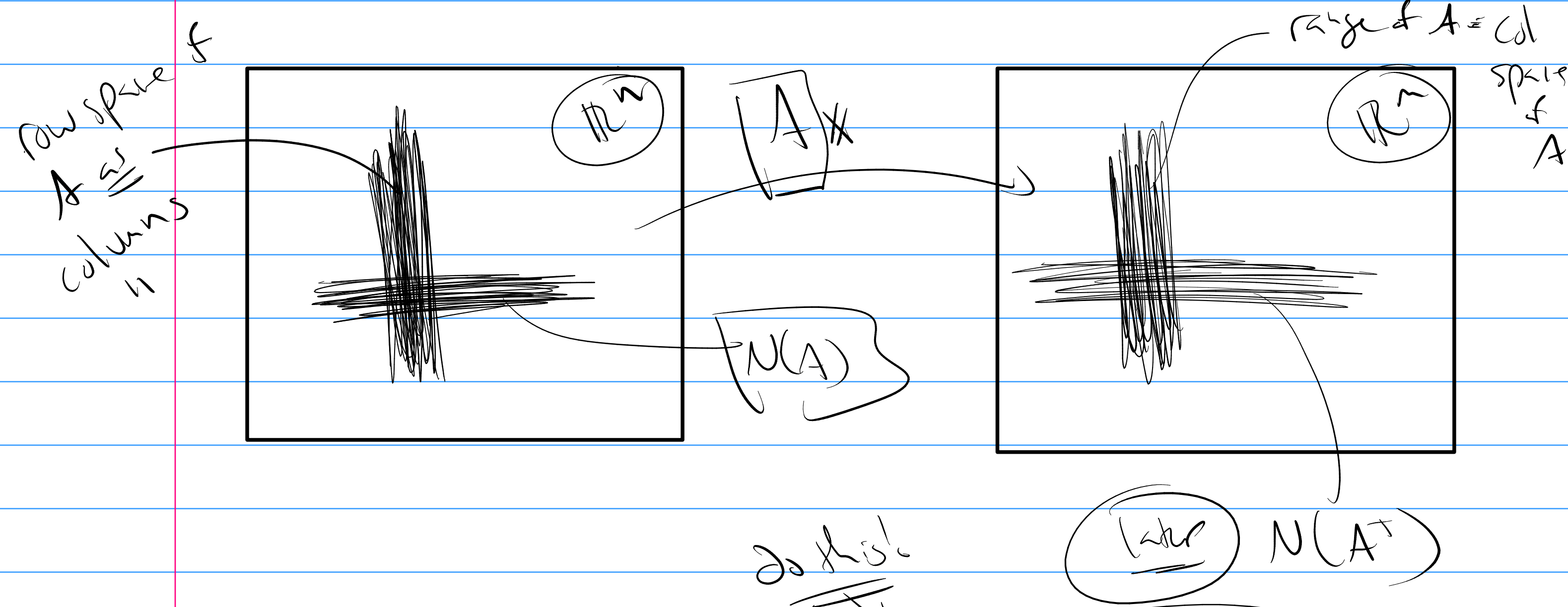
college Alg. example

function

$y = x^2 - 1$







do this!

Say $A = \begin{bmatrix} 1 & -2 & -1 & -1 \\ 2 & -4 & 1 & 4 \\ 3 & -6 & -2 & -1 \end{bmatrix} \xrightarrow{\text{row ops}} \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$

lead | lead

$a_4 = 1a_1 + 2a_3$

$a_2 = -2a_1$

3.6 Facts (1) dep. eqns

$a_2 = -2a_1$

$a_4 = 1a_1 + 2a_3$

(2) col space of A (linear combo of a_i)

$\text{span}(a_1, a_3) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}\right)$

$$\text{call } C = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}_{3\times 2} \leftarrow$$

(3) rowspace of A (linear combos of $\vec{a}_i$)

rowspace of A = span([1 -2 0 1], [0 0 1 2])

$$R = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}_{2\times 4}$$

$\boxed{3\times 4}$

(4) find N(A) solve $\underline{[A \mid 0]}$

(5) rank(A) = # of lead var
nullity(A) = # of free var

(6) Later (find $N(A^T)$)

(7) **check!** $CR = ?$

Exam 2

(1) 3.1 Vector Space?

(2) (3) 3.2 Subspace?

(4) (5) 3.3 check lin. ind?

(6)
(7) } 3.4 Basis / dimension

(8)
(9)] 3.5 change of basis

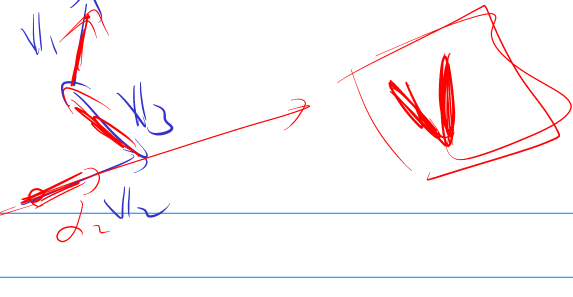
(10)
(11)) 3.6 $A \overset{\text{row ops}}{\rightsquigarrow} U$ (red. row ech.)

col space?
row space?
dep. eqn's?
rank?
nullity?

N(A)?

CR

$\mathrm{span}(\mathbb{V}_1, \mathbb{V}_2, \mathbb{V}_3)$



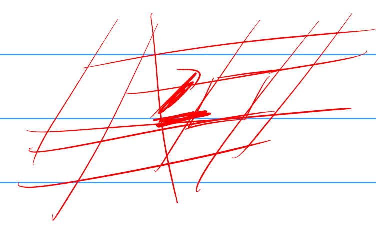
= set of all $\mathbb{V}$ such that

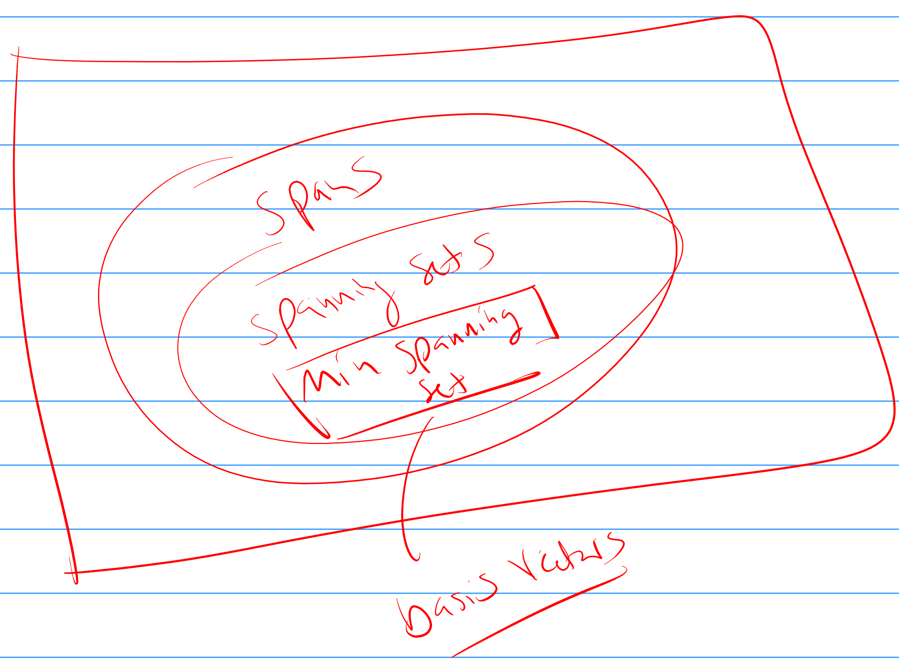
$$\mathbb{V} = \alpha_1 \mathbb{V}_1 + \alpha_2 \mathbb{V}_2 + \alpha_3 \mathbb{V}_3$$

for $\boxed{\text{any}}$ $\alpha_1, \alpha_2, \alpha_3$

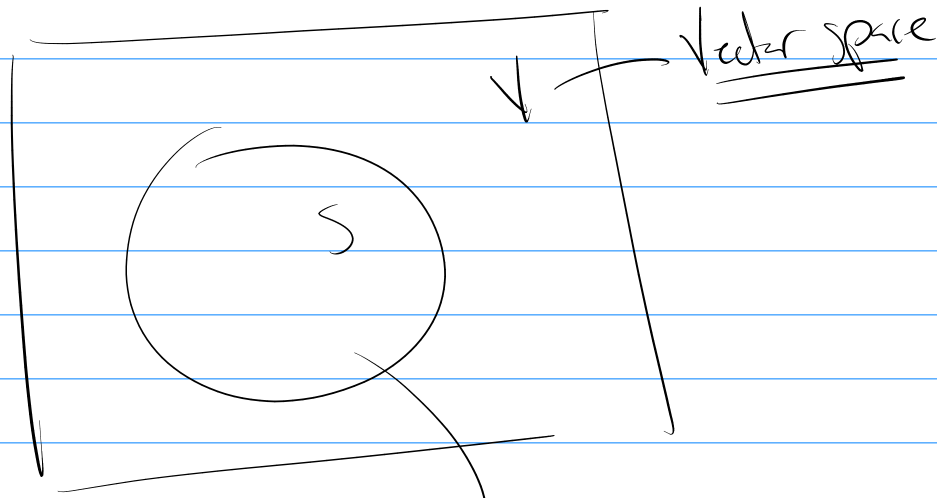
$\mathrm{span}\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \cancel{\begin{bmatrix}2\\1\end{bmatrix}} \right)$ spanning for $\mathbb{R}^2$

basis





getting to basis?



$S = \text{span}(v_1, v_2, \dots, v_k)$

(1) check to see if $\text{span}(v_i) = V$

does $\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_k v_k$ go everywhere?

Solve $\alpha_1 v_1 + \dots + \alpha_k v_k = \left[\text{everywhere variable}\right]$

solution? $\Rightarrow$ $\text{span}(v_i) = V$

call $v_1, v_2, \dots, v_k$ are spanning V

(2) Have spanning? are they all independent?

Say $v_1, v_2, \dots, v_k$

Determine which are ind. / dep

$v_1, v_2, \dots, v_\ell$ $\}$ $v_{\ell+1}, \dots, v_k$

$$\mathrm{Span}(\underset{\underline{k\text{-vectors}}}{\text{all}}) = \mathrm{Span}(\underset{\underline{\ell\text{-vectors}}}{\text{independent}})$$

basis b/c minimal spanning set

$$\underline{\underline{\dim(V) = \ell}}$$