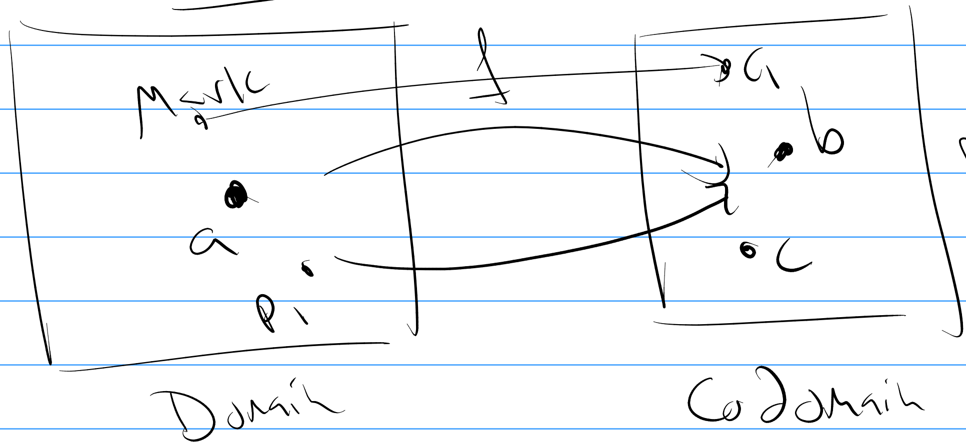
Math 511

Functions! (Calculus?)

preimage, image: $a \xrightarrow{f} b$ or $f(a)=b$

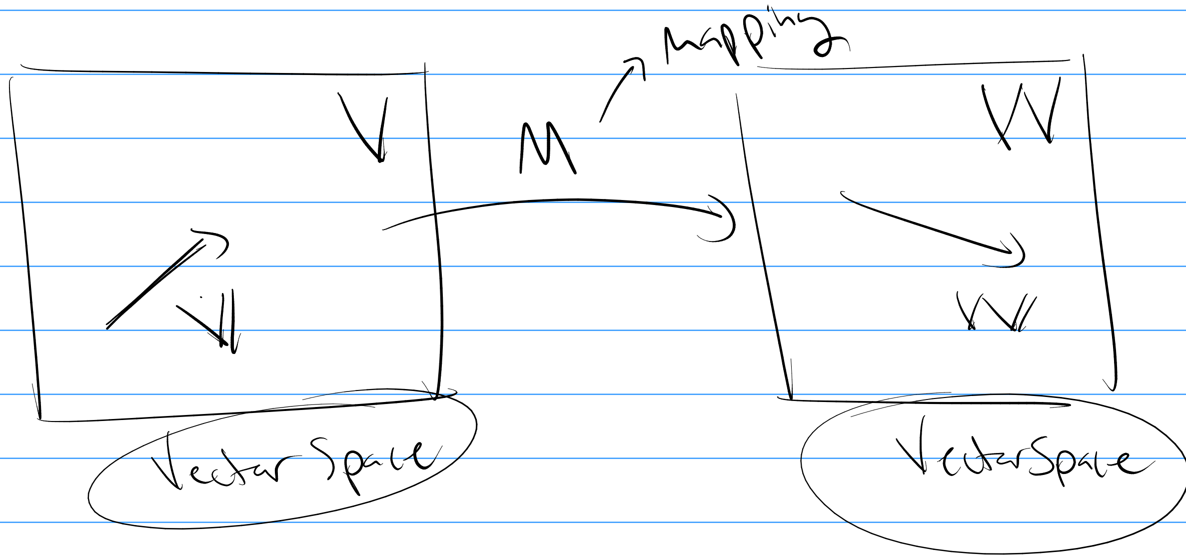
Rev



Def: f is a function iff

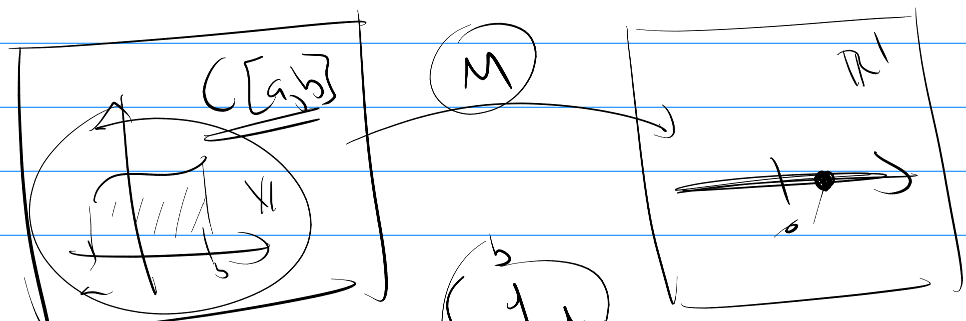
1. everyone in domain maps to the codomain
2. each element in domain is used in only one mapping

Chg

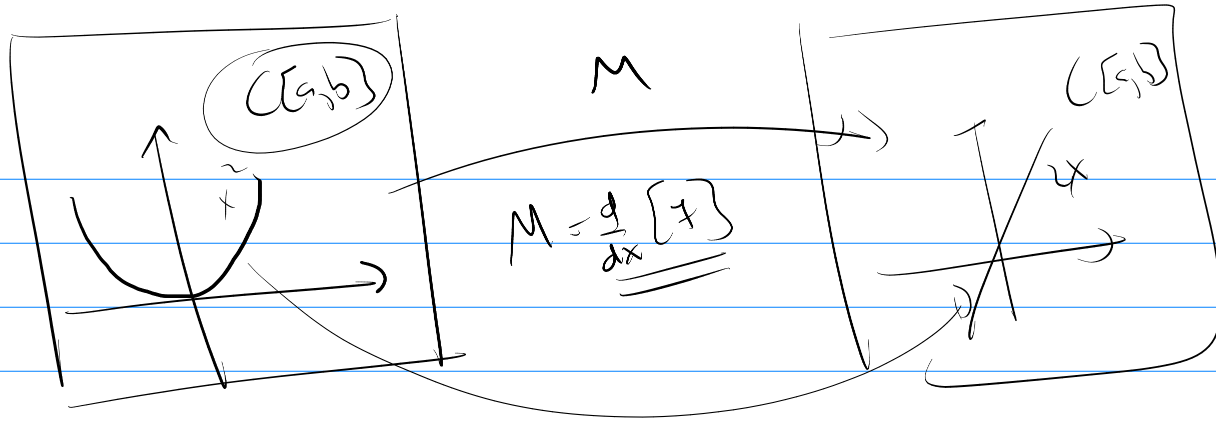


$M(V1) = W1$

ex



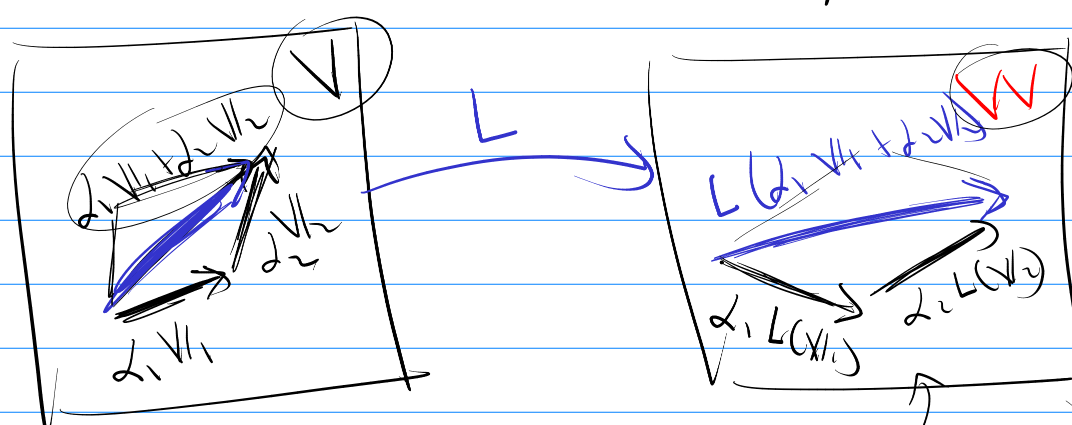
Mapping $= \int_a^b f\,dx$



$$\frac{d}{dx}[x^2] = 2x \qquad x^2 \xrightarrow{\text{deriv}} 2x$$

4.1 Linear Transform

Linear Transform is a mapping from V to W that has the following property..



V, W

check

$$\boxed{L(\alpha_1 v_1 + \alpha_2 v_2) = \alpha_1 L(v_1) + \alpha_2 L(v_2)}$$

vs

check:

1) $L(\alpha_1 v_1) = \alpha_1 L(v_1)$

2) $L(v_1 + v_2) = L(v_1) + L(v_2)$

(ex) is

$L(f) = A$ is $\int_a^b f(x)\,dx = A$

is this a linear transform?

Test #1 Check $L(\alpha_1 V_1 + \alpha_2 V_2) = \alpha_1 L(V_1) + \alpha_2 L(V_2)$

f_1 f_2 f_1 f_2

left: $\int_a^b (\alpha_1 f_1 + \alpha_2 f_2)\,dx = \alpha_1 \int_a^b f_1\,dx + \alpha_2 \int_a^b f_2\,dy$

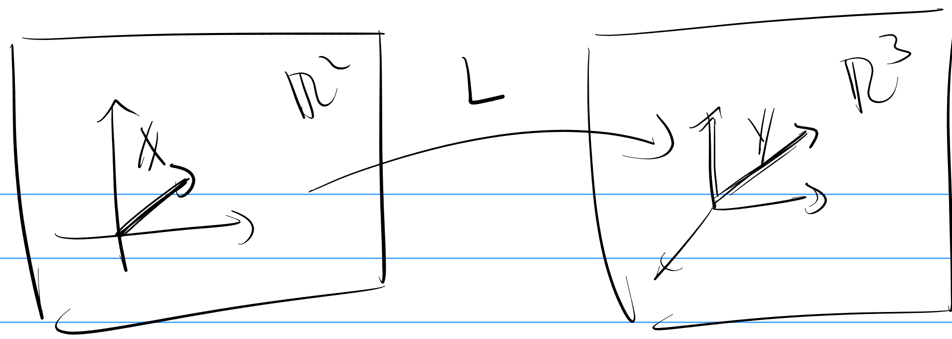
right: $\alpha_1 \int_a^b f_1\,dx + \alpha_2 \int_a^b f_2\,dx$ Same!!

$\int_a^b f(x)\,dx = A$

or $L(f) = A$

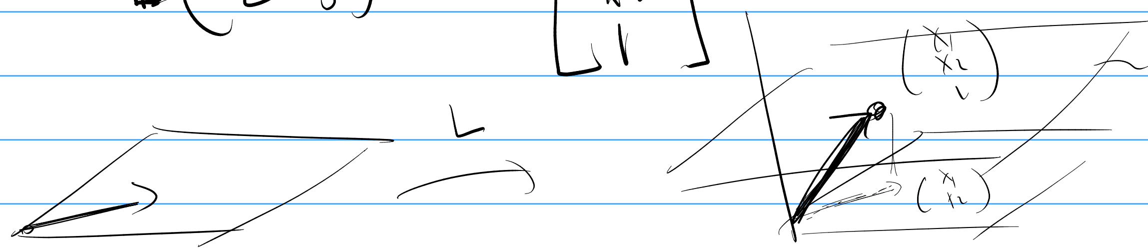
is a linear transform.

6 a



$$L(\mathbb{X}) = (x_1, x_2, 1)^T$$

$$L\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}x_1\\x_2\\1\end{bmatrix}$$



2 step check!

(1) $$L(\overset{2D}{\alpha_1 \mathbb{V}_1}) = \overset{3D}{\alpha_1 L(\mathbb{V}_1)}$$

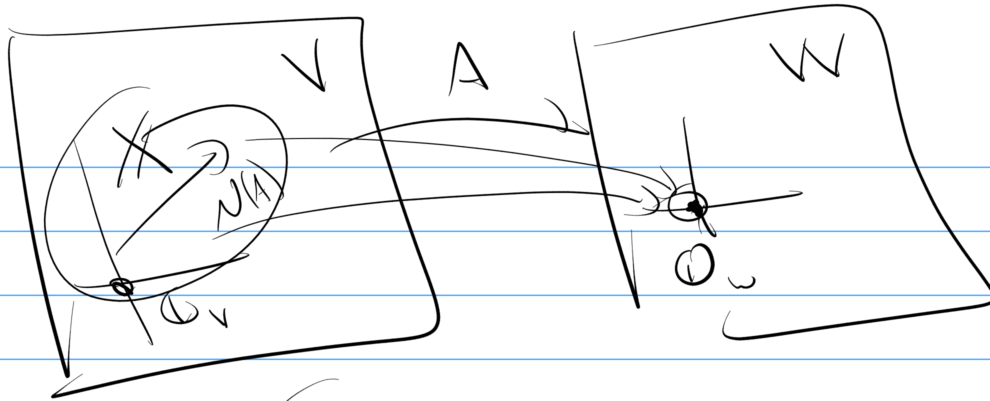
(2) $$L(\underset{2D}{\mathbb{V}_1 + \mathbb{V}_2}) = \underset{3D}{L(\mathbb{V}_1)} + L(\mathbb{V}_2)$$

(1) $$L\left(\begin{bmatrix}\alpha_1 x_1\\ \alpha_1 x_2\end{bmatrix}\right) = \begin{bmatrix}\alpha_1 x_1\\ \alpha_1 x_2\\ 1\end{bmatrix}$$ left — not equal!

$$\alpha_1 L\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \alpha_1 \begin{bmatrix}x_1\\x_2\\1\end{bmatrix} = \begin{bmatrix}\alpha_1 x_1\\ \alpha_1 x_2\\ \alpha_1\end{bmatrix}$$ right

No — it is not a lin. tran

Note:



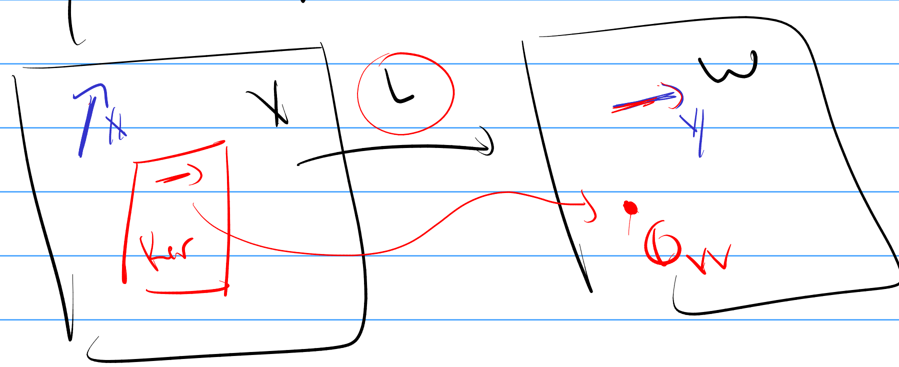
Solve $Ax = 0_w$ homogeneous system

→ find $N(A)$

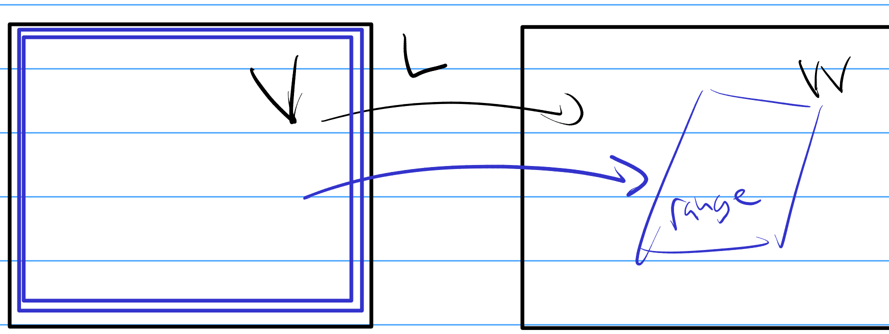
col space of A = range of A

b/c $Ax = x_1 a_1 + x_2 a_2 + \dots + x_n a_n$

So for any $L: V \to W$



Kernel $\mathrm{Ker}(L) = \{ v \in V \mid L(v) = 0_W \}$



range of L: $L(V) = \{ w \in W \mid \text{there is a } v \in V,\ L(v) = w \}$