

Math 511

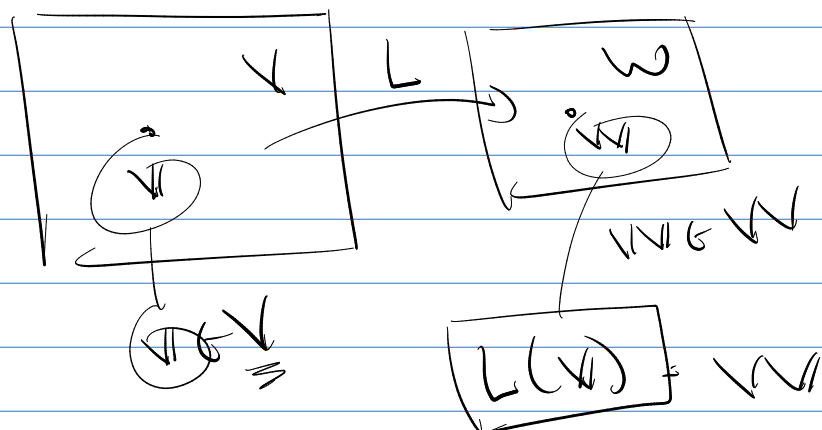
Q3 (4.1)

Linear Transform from V to W ($L: V \rightarrow W$)

Preimage of y

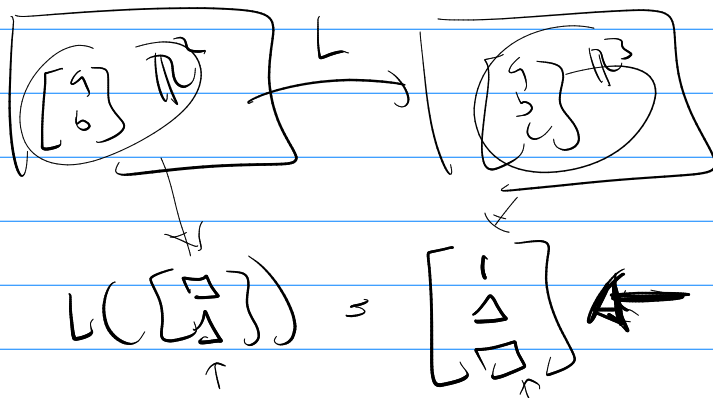
$x \rightarrow y$ image of x

$$y = f(x)$$



$$\textcircled{1} \rightarrow L(\alpha_1 v_1 + \alpha_2 v_2) = \alpha_1 L(v_1) + \alpha_2 L(v_2)$$

$\textcircled{2}$



$$L\left(\begin{bmatrix} 9 \\ 6 \end{bmatrix}\right) = \begin{bmatrix} 9 \\ 3 \\ 2 \end{bmatrix}$$

Linear Transform

2 step

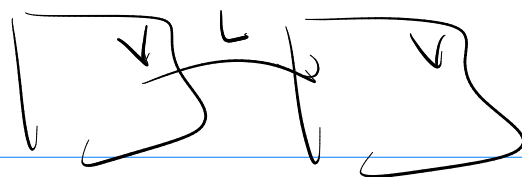
$$\textcircled{1} \quad L(\alpha_1 v_1) = \alpha_1 L(v_1)$$

$$L(\alpha_1 [x_1]) = L\left(\begin{bmatrix} \alpha_1 x_1 \end{bmatrix}\right) = \begin{bmatrix} \alpha_1 x_1 \end{bmatrix}$$

$$\alpha_1 L\left(\begin{bmatrix} x_1 \end{bmatrix}\right) = \alpha_1 \begin{bmatrix} x_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 x_1 \end{bmatrix}$$

Not Lin. trans

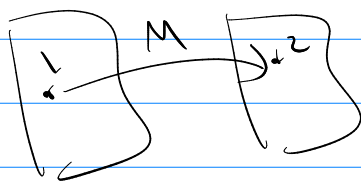
Linear Operator on V



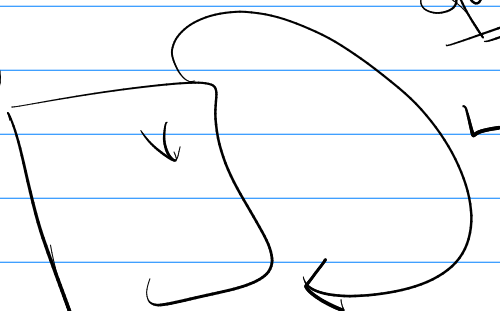
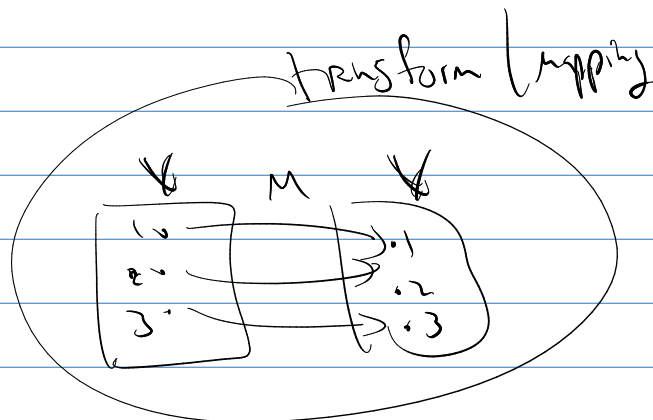
Linear transform from V to V

$$L: V \rightarrow V$$

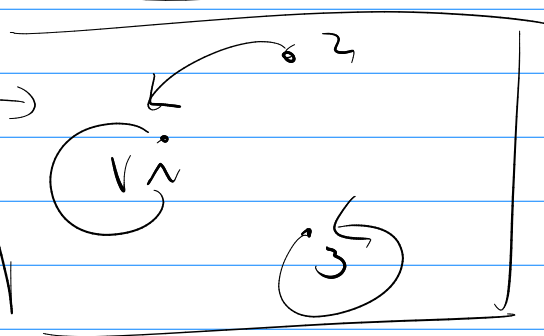
Concept



$$z = M(v)$$

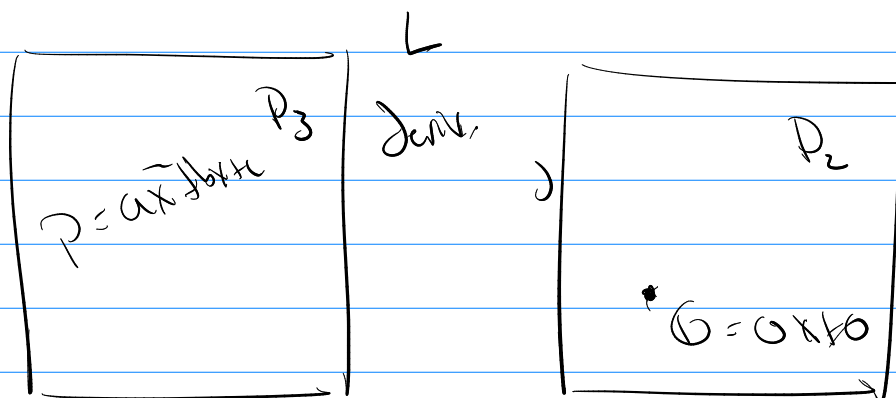


operator



$$\text{Ker}(L) = \{ v \in V \mid L(v) = 0_V \}$$

(ex)

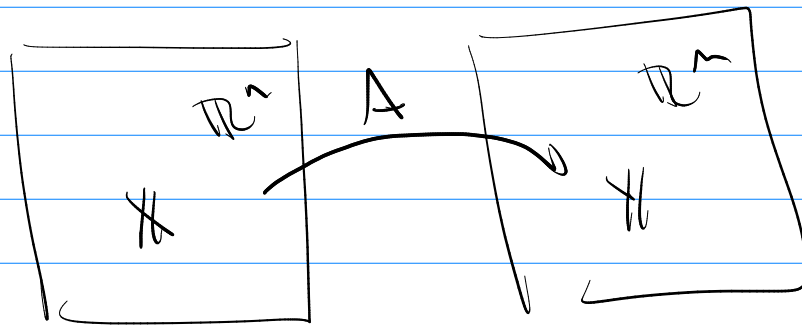


$$L(\underbrace{ax^2 + bx + c}_{P_3}) = \underbrace{2ax + b}_{P_2}$$

$$\text{Dom} \left(\begin{bmatrix} ax^2 + bx + c \\ 0 \end{bmatrix} \right) = \{x \mid 0 \in P_2\}$$

$$\begin{aligned} \ker(L) &= \{ax^2 + bx + c \mid c \text{ is any real}\} \\ &= P_1 \end{aligned}$$

4.2 From 4.1 we know any matrix mult. $\boxed{B}$ is linear trans



$$\underset{m \times n}{A} \underset{n \times 1}{x} = \underset{1 \times 1}{y}$$

$$\text{and } A(\alpha_1 x_1 + \alpha_2 x_2) = \alpha_1 A x_1 + \alpha_2 A x_2$$

$\uparrow \qquad \qquad \qquad \uparrow \qquad \qquad \uparrow$

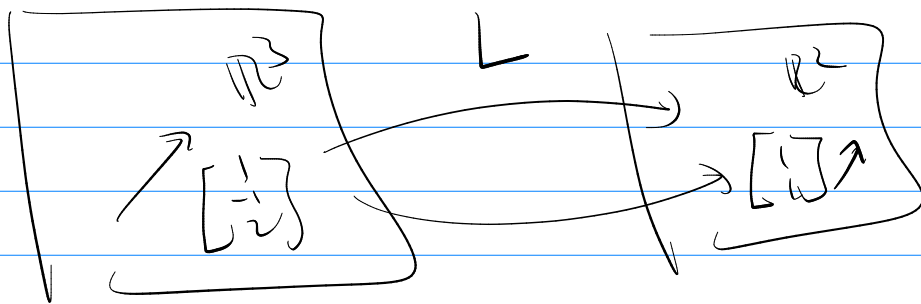
So $Ax = L(x) = y \quad B \in \underline{\underline{\text{linear transform}}}$

Can we go transform into a matrix multiply?

(14) given $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2 + x_3 \\ x_1 \end{bmatrix}$ (15) Linear trans.

$\mathbb{R}^3 \quad \quad \quad \mathbb{R}^2$

so $L\left(\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -1 + 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



Linear transform (1) $L(2v_1) = 2L(v_1)$

(16) $L\left(2v_1\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}\right) = L\left(\begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2(b+c) \\ 2a \end{bmatrix}$

$2L\left(\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}\right) = 2L\begin{bmatrix} b+c \\ a \end{bmatrix} = \begin{bmatrix} 2(b+c) \\ 2a \end{bmatrix}$

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \begin{pmatrix} d \\ e \\ f \end{pmatrix}$

(2) $L(v_1 + v_2) = L(v_1) + L(v_2)$

$L\left(\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix}\right) = \begin{bmatrix} b+e+c+f \\ a+d \end{bmatrix}$

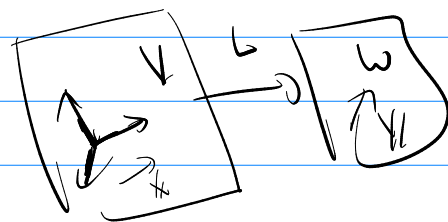
$L\left(\begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}\right) + L\left(\begin{bmatrix} d \\ e \\ f \end{bmatrix}\right) = \begin{bmatrix} b+c \\ a \end{bmatrix} + \begin{bmatrix} e+f \\ d \end{bmatrix} = \begin{bmatrix} b+e+c+f \\ a+d \end{bmatrix}$

true

to find a matrix A such that

$$L(X) = Y$$

is also $A X = Y$



use standard coord for both V, W
standard basis

$$X = [X]_E$$

$$Y = [Y]_E$$

$$A [X]_E = [Y]_E$$

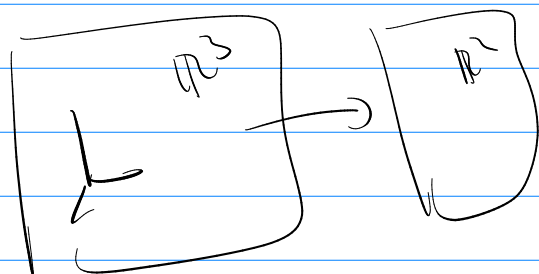
pf

$$A = [L(e_1) \quad L(e_2) \quad \dots \quad L(e_n)]$$

$$n = \dim(V)$$

standard for V

so $L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$



$A = ?$

$$A = [L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) \quad L\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) \quad L\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)]$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

So $L\left(\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b/c)

Now $L\left(\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

2×3 3×1

So $L(x) = A x$

Now $A [x]_{\mathcal{E}} = [y]_{\mathcal{E}}$

$\begin{bmatrix} 6 \\ 5 \end{bmatrix} \xrightarrow{\text{Deriv}} \begin{bmatrix} 0+bx+cx^2 \\ 1+bx \end{bmatrix}$

$\begin{matrix} \text{Standard} & 1, 0, x, x^2 \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix} \end{matrix}$

$\begin{matrix} \text{Deriv (arbitrary)} \\ = 0+bx+cx^2 \\ = b+2c \end{matrix}$

$\begin{matrix} \text{Standard } 1, x \\ \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 6 \\ 1 \end{bmatrix} \end{matrix}$

$L\left(\begin{bmatrix} 6 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

$A = \begin{bmatrix} L(1) & L(0) & L(x) \end{bmatrix}$

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Deriv of any $ax+bx^2$

So $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$