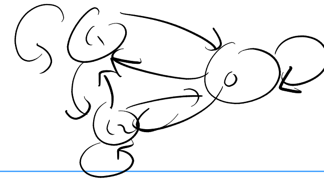
Math 511



4.2 #17 $\quad L: V \to V \quad$ basis $\underline{v_1, v_2, \ldots, v_n} \quad \dim(V) = n$

$$Ax = y \qquad A[x]_V = [y]_V = L(x)$$

$$L^m(x) = L(L(L(L(L(L(x))))))$$

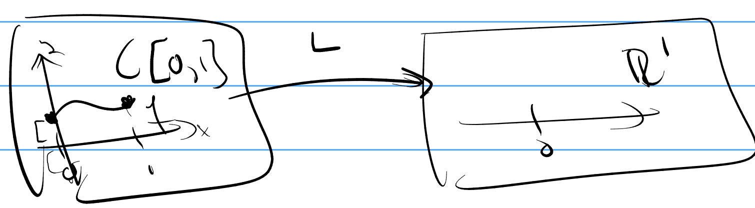
operator $\quad$ m-times.

$$= L(\cdots L(L(A[x]_V)))$$

$$= L(\cdots L(AA[x]_V))$$

$$= \underbrace{A \cdot A A}_{m\text{-factors}} [x]_V = A^m [x]_V$$

4.1 (11b)



$$L(f) = |f(0)| \quad \text{functions}$$

Linear Transform? $\quad \underbrace{L(\alpha_1 v_1 + \alpha_2 v_2)}_{\mathbb{R}^1} = \alpha_1 \underbrace{L(v_1)}_{\mathbb{R}^1} + \alpha_2 \underbrace{L(v_2)}_{\mathbb{R}^1}$

a) $L(\alpha_1 f_1 + \alpha_2 f_2) = |\alpha_1 f_1(0) + \alpha_2 f_2(0)|$

b) $\alpha_1 L(f_1) + \alpha_2 L(f_2) = \alpha_1 |f_1(0)| + \alpha_2 |f_2(0)|$ $\quad \}\ \neq$

SU $\boxed{\text{NO}}$

$\boxed{V} \xrightarrow{L} W$

(4.2) $L(x) = y$ $\quad$ $\dim(V) = n$

<u>standard coord</u>

$$A[x]_E = [y]_E$$

$$A = \begin{bmatrix} L(e_1) & L(e_2) & \cdots & L(e_n) \end{bmatrix}$$

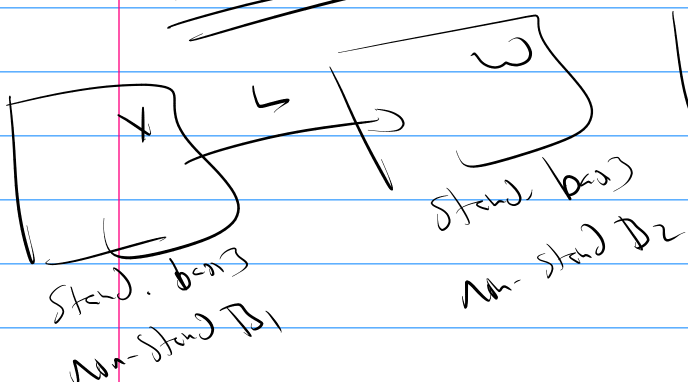
<u>Non-Standard</u> (Know <u>change of basis</u>)

① find A, standard matrix form of L

$$\rightarrow \boxed{A}[x]_E = [y]_E$$

② Any other basis? $B_1, B_2 \ldots$

<u>do transform</u>



$$\underbrace{B_2^{-1}\left(\underbrace{A\,(B_1 \overset{\uparrow}{[x]_{B_1}})}_{[x]_E}\right)}_{[y]_E} = [y]_{B_2}$$

so $\boxed{S = B_2^{-1} A B_1}$ transform using basis B_1 to basis B_2 coordinates.

<u>Mixture of idea!</u>

$$L(c_1 b_1 + c_2 b_2 + c_3 b_3) = \begin{bmatrix} c_1 + c_2 \\ c_3 \end{bmatrix}_E$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}_B$$

$$\underline{AB} = \left[\underset{\mathbb{b}_1}{L\begin{pmatrix}1\\0\\0\end{pmatrix}} \quad \underset{\mathbb{b}_2}{L\begin{pmatrix}0\\1\\0\end{pmatrix}} \quad \underset{\mathbb{b}_3}{L\begin{pmatrix}0\\0\\1\end{pmatrix}} \right] = \begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

4.3 Focus on Linear Operators $L: V \to V$

<u>Basis?</u> $E, B_1, B_2, \ldots$

<u>non-standard</u>

(1) A : standard $\quad A[x]_E = [y]_E$

$\boxed{\text{is}}$ L

(2) L as a matrix in non-standard -- like B_1 ?

$$\underbrace{\boxed{B_1^{-1} A B_1}}_{S}[x]_{B_1}$$

$\boxed{\text{is}}$ L

$$S[x]_{B_1} = [y]_{B_1}$$

S $\boxed{\text{is}}$ L

A $\boxed{\text{is}}$ L

$$S = B_1^{-1} A B_1$$

$$B_1 S B_1^{-1} = A$$

call S, A similar

Def

if $$M_1 = T^{-1} \cdot M_2 \cdot T$$

call M_1, M_2 similar