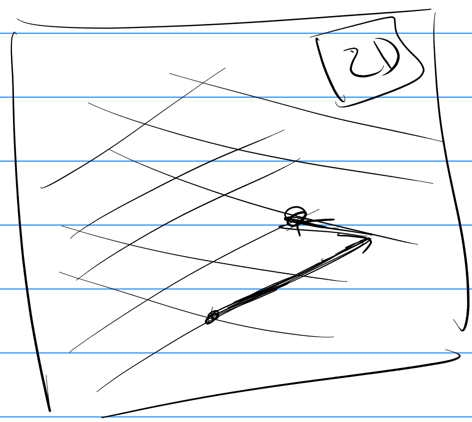
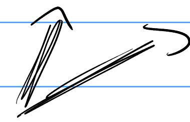
Math 511

Vector spaces

physical idea (only in 2D, 3D)



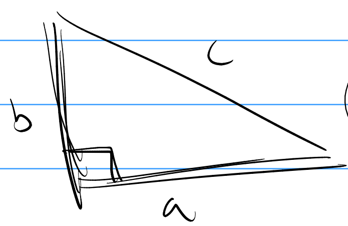


true vectors

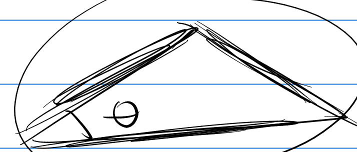
Geometry

middle?

ex



$a^2+b^2=c^2$



$\cos\theta$

perpendicular

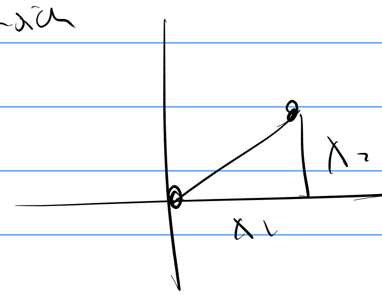
length

parallel

$c=(a^2+b^2)^{1/2}$

5-1 study 2D, 3D, $\mathbb{R}^2$, $\mathbb{R}^3$, ..., n-Dimension $\mathbb{R}^n$

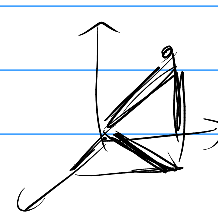
"known"



2D length $=(x_1^2+x_2^2)^{1/2}$

3D length $=(x_1^2+x_2^2+x_3^2)^{1/2}$

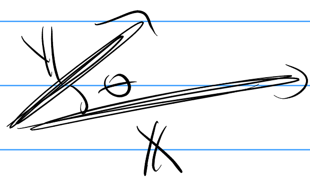
ND length $=(x_1^2+x_2^2+\dots+x_n^2)^{1/2}$



N-D length $= (\mathbf{x}^T\mathbf{x})^{1/2} = (x_1\, x_2 \cdots x_n)\begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{pmatrix}$

use $\boxed{\text{length}^2 = \underbrace{\mathbf{x}^T\mathbf{x}}_{\text{Scalar product.}}}$

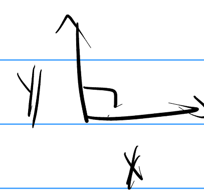
Notation: $\|\mathbf{x}\|^2 = \mathbf{x}^T\mathbf{x}$



$\mathbf{x}^T\mathbf{y} = \|\mathbf{x}\|\|\mathbf{y}\|\cos\theta$

b/c $-1 \le \dfrac{\mathbf{x}^T\mathbf{y}}{\|\mathbf{x}\|\|\mathbf{y}\|} \le 1$

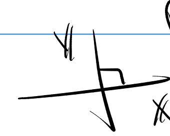
so $\mathbf{x}^T\mathbf{y} = 0$ only if



$$\boxed{\frac{\mathbf{x}^T\mathbf{y}}{\|\mathbf{x}\|\|\mathbf{y}\|} = \cos\theta}$$

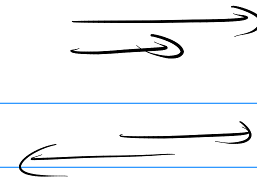
and $0 \le |\mathbf{x}^T\mathbf{y}| \le \|\mathbf{x}\|\,\|\mathbf{y}\|$

$|\mathbf{x}^T\mathbf{y}| = 0$ if



perpendicular

$$|x^T y| = \|x\| \|y\| \quad \text{if}$$



parallel

physical (cos)

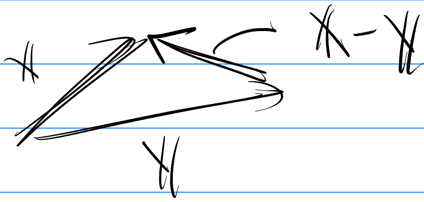
2D, 3D, ... ND $\quad \mathbb{R}^n$

$$\|x\|^2 = x^T x$$

$$\cos\theta = \frac{x^T y}{\|x\| \|y\|}$$

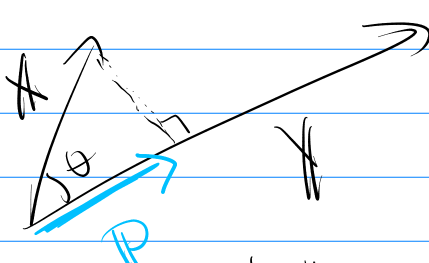
$$-1 \le \frac{x^T y}{\|x\| \|y\|} \le 1$$

$$|x^T y| \le \|x\| \|y\|$$



$\|x - y\|$ called distance between x, y

cos



P = projection of x onto y

$\|P\|$ = scalar proj of x onto y

$$= \frac{x^T y}{\|y\|}$$

$$\mathbb{P} = \frac{x^T v}{\|v\|} \cdot \left(\frac{v}{\|v\|} \right)$$ direction of v (unit vector)

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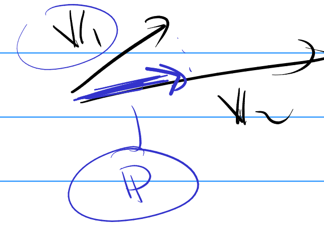
① Scalar proj of v_1 onto v_2



$$= \frac{v_1^T v_2}{\|v_2\|}$$

② Vector proj of v_1 onto v_2

$$= \frac{v_1^T v_2}{v_2^T v_2} v_2$$

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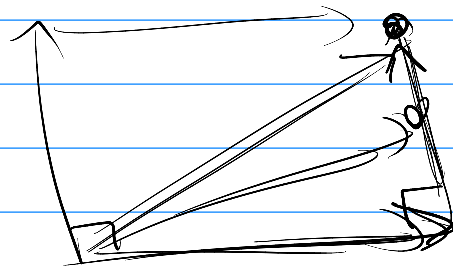
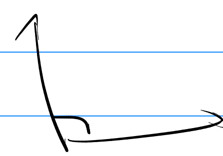
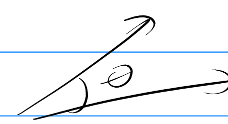
orthogonal



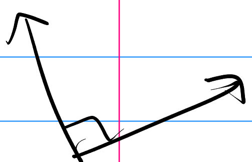


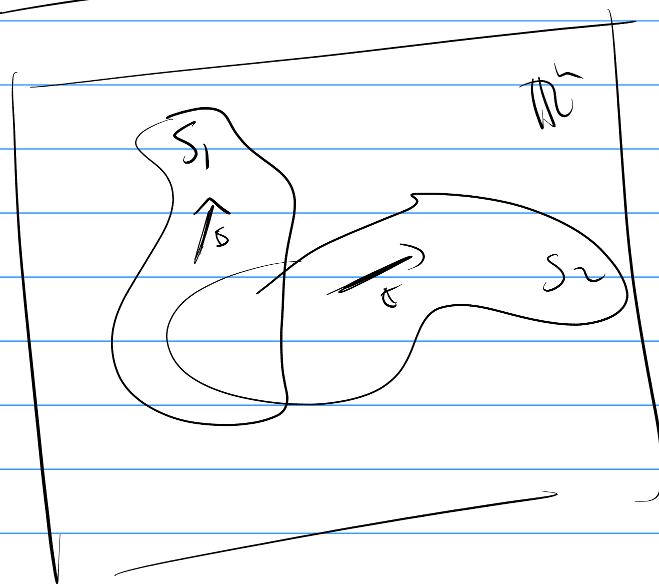


$1 + x$, $x^2 - 2$

$a + bx + cx^2$

5.3 orthogonal subspaces





if for any $s \in S_1$

any $t \in S_2$

$\underbrace{s^T t = 0}_{\text{all orthogonal}}$

$\Downarrow$

call S_1, S_2 to be orthogonal