Math 511

4.3 #6 **Similarity:** $A = S^{-1} B S$

call A, B to be similar

Part

$\dim(C[a,b]) = \infty$ (hard!) so we normally do [all] $C[a,b]$

we do a part of it

(ex) 4.3 #6 $S = \text{Span}(1, e^{x}, e^{-x}) = B$

Facts

Calculus hyperbolic functions

$B: \{1, e^{x}, e^{-x}\}$

$$\cosh(x) = \frac{1}{2}e^{x} + \frac{1}{2}e^{-x} = \frac{e^{x}+e^{-x}}{2}$$

$$\sinh(x) = \frac{1}{2}e^{x} - \frac{1}{2}e^{-x} = \frac{e^{x}-e^{-x}}{2}$$

so $\cosh(x) = 0(1) + \frac{1}{2}e^{x} + \frac{1}{2}e^{-x} = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \end{bmatrix}_B$

$\sinh(x) = 0(1) + \frac{1}{2}e^{x} - \frac{1}{2}e^{-x} = \begin{bmatrix} 0 & 1/2 & -1/2 \end{bmatrix}^T_B$

if I use $\{1, e^x, e^{-x}\}$ as my standard.

$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ where $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \boxed{a(1) + b(e^x) + c(e^{-x})}$

so $\{1, \cosh(x), \sinh(x)\}$

$B_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{bmatrix}$

b/c $1 = 1(1) + 0(e^x) + 0(e^{-x})$

$\cosh x = \boxed{0(1) + \frac{1}{2}(e^x) + \frac{1}{2}(e^{-x})}$

$\sinh x = 0(1) + \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

Operator : take the derivative.

(a) change of coord. from standard to B_1

(*) Know.

$$[v]_E = B_1 [v]_{B_1}$$

$$[v]_{B_1} = B_1^{-1} [v]_E$$

in (a) $S = B_1^{-1}$

(b) B (operator as a matrix in stand)

$B = (L(P_1) \;\; L(P_2) \;\; L(P_3))$

take the deriv.

for this (c) →

problem they are calling standard rep. B

but (b) is supposed to be in B_1!!

$$[L(v)]_{B_1} = \boxed{B_1^{-1}\ B\ B_1}\ [v]_{B_1}$$

A in basis of B_1

non-stand rep. of operator.

$A = B_1^{-1} B B_1$ or

$$B = \underbrace{(B_1)}_{S^{-1}} A \underbrace{(B_1^{-1})}_{S}$$

S – part (a)

S^{-1} part (a)

---

know

change of coord. for a basis

① $[v]_E = B_1 [v]_{B_1}$

② $[v]_{B_1} = B_1^{-1} [v]_E$

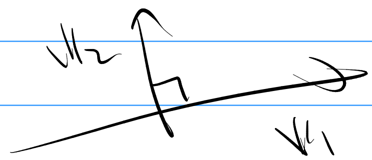
③ Standard matrix of an operator

$\to Q = [L(e_1)\ \ L(e_2)\ \cdots\ L(e_n)]$

④ other basis (B_1)

$R = B_1^{-1} Q B_1$

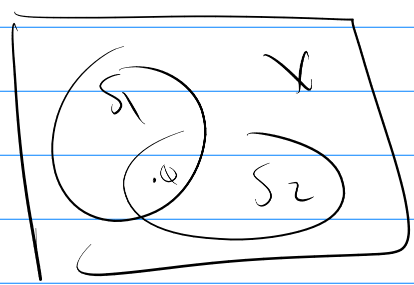
Def: <u>orthogonal</u>: original def is between one vector and second vector.



def: $v_1^T v_2 = 0$

call v_1, v_2 to be <u>orthogonal</u>

Def: S_1, S_2 are subspaces of V, vector space



and I see <u>all</u> $v \in S_1$ are orthogonal to <u>all</u> $w \in S_2$

ex $\mathbb{R}^4$ (4D)

$$S_1 = \text{Span}\left(\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right) \quad \text{2D (plane)}$$

$$S_2 = \text{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right) \quad \text{1D (line)}$$

<u>notice:</u> (using basis)

$$\begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

so all $v \in S_1$ $\perp$ to all $w \in S_2$

call $S_1 \perp S_2$

$\boxed{\text{Def}}$ Orthogonal complement

given S a subspace of V

orthogonal complement of S = $S^{\perp}$

$$S^{\perp} = \{ \text{all vectors in } V \text{ that are } \perp \text{ to all } v \in S \}$$

<u>Note:</u> $\dim(S) + \dim(S^{\perp}) = \dim(V)$

Use this to make $S, S^{\perp}$

ex $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ spans $\mathbb{R}^4$ (basis of $\mathbb{R}^4$)

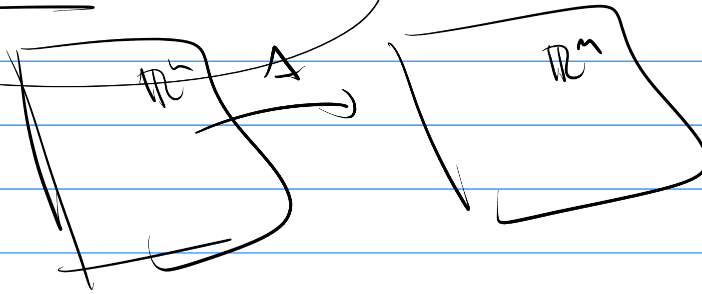
ex $S_1 = \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right)$

$S_1^{\perp}$ = <u>2D</u> Need two vectors that are both $\perp$ to both

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

but there are two fundamental subspaces for $\mathbb{R}^n$ and $\mathbb{R}^m$.

given

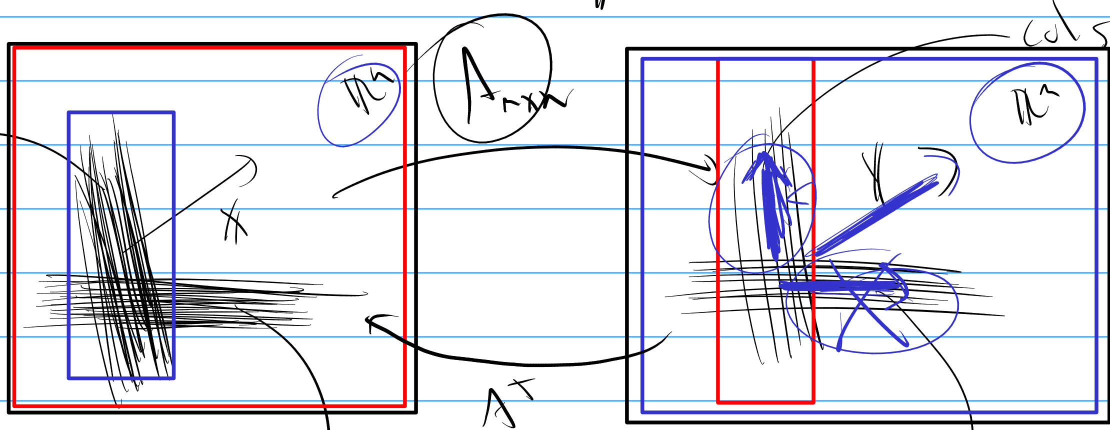


fundamental orthogonal subspaces of $\mathbb{R}^n$, $\mathbb{R}^m$

$Ax = y$ $\quad x \in \mathbb{R}^n$ $\quad y \in \mathbb{R}^m$

col space of A^T = range of A^T = row space of A written as col vectors

col space of A = range of A



$Ax = y$

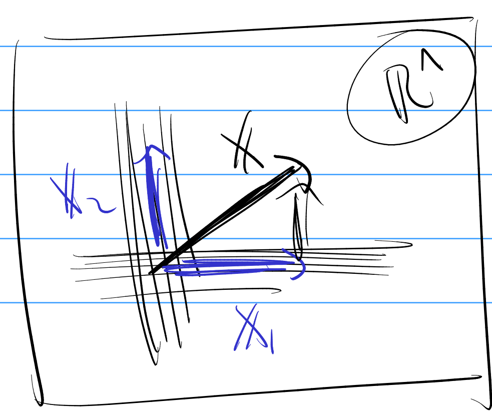
$N(A)$ $\qquad$ $N(A^T)$

Facts!

$N(A) \cup \text{col space } A^T = \mathbb{R}^n$

$N(A^T) \cup \text{col space } A = \mathbb{R}^m$

So?



$\dim(\mathbb{R}^n)$

$x = x_1 + x_2$

$x_1 \in N(A)$

$x_2 \in \text{col space of } A^T$