Math 511

4.3 #6

$$\int x\sin(x^2)\,dx \;\cancel{\longrightarrow}\; -\frac{1}{2}\cos x^2 + C$$

$u = x^2$
$du = 2x\,dx$

$$\frac{1}{2}\int \sin u\,du \longrightarrow -\frac{1}{2}\cos u + C$$

4.3 #6

$$1, \cos^2 x, \sin^2 x \to \begin{bmatrix}1&0&0\\0&1/2&1/2\\0&1/2&-1/2\end{bmatrix}$$

Deriv

$$[y']_{1,\cos^2,\sin^2} = \begin{bmatrix}1&0&0\\0&1/2&1/2\\0&1/2&-1/2\end{bmatrix}\begin{bmatrix}0&0&0\\0&1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1/2&1/2\\0&1/2&-1/2\end{bmatrix}[y]_{1,\cos^2 x,\sin^2 x}$$

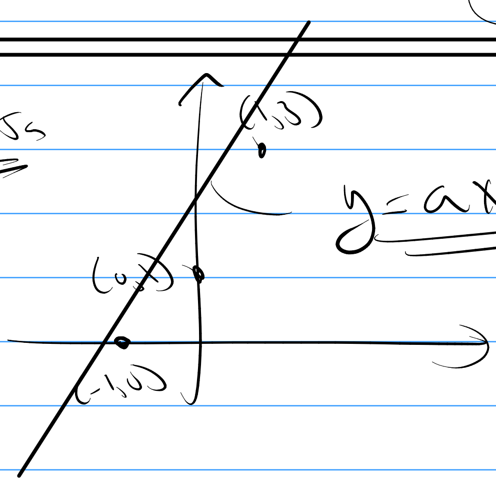
6a. S

6b. A

,(3,5)

6c. D

5.3 5a



$y = ax + b$

through points?

$a(-1) + b = 0$
$a(0) + b = 1$
$a(1) + b = 3$
$a(2) + b = 9$

$$\begin{bmatrix}-1&1\\0&1\\1&1\\2&1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}0\\1\\3\\9\end{bmatrix}$$

V c y

no sol?

(now what?)

$V\mathbf{a} = \mathbf{y}$ no soln?

Solve: $V^T V \mathbf{a} = V^T \mathbf{y} \rightsquigarrow$ soln $\mathbf{a} = (V^T V)^{-1}(V^T \mathbf{y})$

called least square soln.

$$\mathbf{a} = \left(\begin{bmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \right)^{-1} \left(\begin{bmatrix} -1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 3 \\ 2 \end{bmatrix} \right)$$

$$\mathbf{a} = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 21 \\ 13 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \quad \&$$

$y = ax + b$

In general: Fit $P_1, P_2, \ldots, P_n$ (n-points)

(x_1, y_1) (x_2, y_2)

to a k degree poly.

$$P = a_k x^k + a_{k-1} x^{k-1} + \cdots + a_1 x + a_0$$

then V

$$V = \begin{bmatrix} x_1^k & x_1^{k-1} & \cdots & x_1^0 \\ x_2 & x_2 & & x_2 \\ x_3 & x_3 & & x_3 \\ \vdots & \vdots & & \vdots \\ x_n & x_n & & x_n \end{bmatrix}$$

Solve $V\mathbf{a} = \mathbf{y}$ (no soln)

$\downarrow$

Solve $V^T V \mathbf{a} = V^T \mathbf{y}$

$$\boxed{\mathbf{a} = (V^T V)^{-1}(V^T \mathbf{y})}$$

$$V \underbrace{\begin{bmatrix} a_k \\ \vdots \\ a_0 \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{\mathbf{y}}$$

Exam: 11 probs (same as other exams)

$V \to W$

4.1 (1 prob)

a) b) } given a mapping from $V \to W$

Q: is mapping a linear transform $L: V \to W$?

check: $L(\alpha_1 v_1 + \alpha_2 v_2) \stackrel{?}{=} \alpha_1 L(v_1) + \alpha_2 L(v_2)$

yes? → transform and $\mathrm{Ker}(L)$, $\mathrm{Range}(L)$

no? → not

ex Mapping from P_3 to P_2

$$\boxed{ax^2+bx+c} \to \boxed{(a-c)x+b^2}$$

transform? 2 step check

(1) $L(\alpha p) \stackrel{?}{=} \alpha L(p)$

$$L(\alpha a x^2 + \alpha b x + \alpha c) = (\alpha a - \alpha c)x + (\alpha b)^2$$

vs

$$\alpha L(p) = \alpha\left[(a-c)x+b^2\right] = \alpha(a-c)x + \alpha b^2$$

no!

4.2 (2 probs)

$L(\vec{x})$ as $A\vec{x}$ — A is matrix rep of L a linear trans from standard to standard basis

$[\vec{x}]_E \longrightarrow [A\vec{x}]_E$

① given $L: V \to W$ write A for L.

② ex $\text{Anti}(\text{poly})$ (constant = 0)

$p = \begin{bmatrix} \text{const} \\ x \\ x^2 \\ \vdots \end{bmatrix}$ vector

ex $\text{Anti}(3x^2+2) = x^3+2x$ $\quad \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

Standard $1, x, x^2$ $\quad P_3 \xrightarrow{\text{Anti}} P_4 \quad$ Standard $1, x, x^2, x^3$

Anti is the standard matrix?

$$A = [\,L(P_1)\ \ L(P_2)\ \cdots\ L(P_n)\,]$$

L — Anti, P_1 — P_3, result in P_4

$$A = [\,\text{Anti}(1)\ \ \text{Anti}(x)\ \ \text{Anti}(x^2)\,]$$

$= x \qquad = \frac{1}{2}x^2 \qquad \frac{1}{3}x^3$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

4×3

So $\text{Anti}(3x^2+2)$ is $\underset{4\times 3}{A}\underset{3\times 1}{\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}} = \underset{4\times 1}{\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}}$

4.2 (2nd prob) Same idea as 1st of 4.2

but non-standard coord.

Know

$$\boxed{\begin{aligned}[v]_E &= V\,[v]_V \\ [v]_V &= V^{-1}[v]_E\end{aligned}}$$

but

(ex) $[p^{Aint}]_V = \boxed{V^{-1} A V} [p]_V$

(ex) non-stand $(2, 3-x, x^2-1)$ for P_3

$$p = \begin{bmatrix} \text{coeff} \\ x \\ x^2 \\ 1 \end{bmatrix} \qquad V = \begin{bmatrix} 2 & 3 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

leave in non-std.

4.3 (1 prob)

like #6 P_n

5.1 (2 probs) $\mathbb{R}^n$ use $\mathbf{x}^T\mathbf{y}$

(1)
(2) Involve: a) are v_1, v_2 orthogonal?
b) $\cos\theta = ?$ or $\theta = ?$ for v_1, v_2

c) $\|v\|$?
d) proj. of v_1 onto v_2?

5.2 (2 probs)

2 diff. A's
(1) given A find basis of $R(A), N(A), R(A^T), N(A^T)$
(2) understand how $R(A), N(A), R(A^T), N(A^T)$ are related.

5.3 (1) data fit

5.4 (2 probs)

$\langle \mathbf{x}, \mathbf{y} \rangle$

(1) is the func $\langle \mathbf{x}, \mathbf{y} \rangle$ a inner product?

$\mathbb{R}$

a) No / yes #
b)

A

Matrix Spaces

$\mathbb{R}^{2\times 3}$

(2) v_1, v_2 for $\langle \mathbf{x}, \mathbf{y} \rangle$ in V
orthogonal?
$\cos\theta$?
norm? = "length"
projection

5.1

Q Range(L)

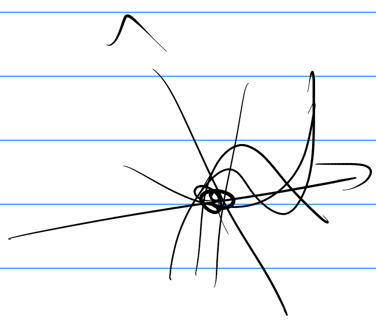
transform $\langle v \rangle$ of V
= as a variable vector

ex Anti (all P_3) don't choose (const of int = 0)

$$\text{Anti}(ax^2+bx+c) = \underbrace{\tfrac{1}{3}ax^3 + \tfrac{1}{2}bx^2 + cx + 0}_{\text{in } P_4}$$

$\text{range}(L) = \text{all } c_1x^3 + c_2x^2 + c_3x$

= any P_4 polynomial with a 0 constant



= any P_4 polynomial that has $x=0$ as a root

= any P_4 polynomial whose graph goes through origin

$$= \text{Span}\left(\begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right)$$