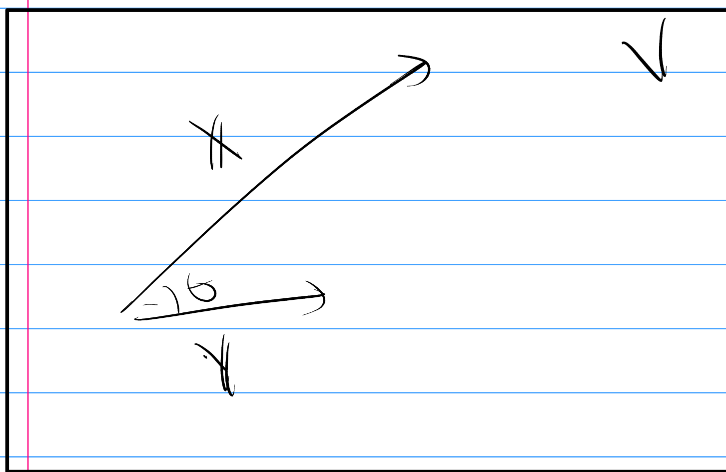
Math 511



$C[a,b] \to x$ are $f(x)$

$\mathbb{R}^n \to x$ is $[x_1, x_2, \ldots x_n]^T$

P_n

$P \to x$ is $p(x)$

and each have $\langle\ ,\ \rangle$ → θ? ; $\|x\|$

$\boxed{x \text{ "IS"}}$

Q? represent x? → coordinates $[x]_{\text{Basis}}$

Finding coord and to use coordinates

Fact #1 orthogonal basis is good.

Fact #2 orthogonal and each vector of basis is length 1 is better!

$B = \{b_1, b_2, \ldots, b_k\}$

and ① $\langle b_i, b_j \rangle = 0$ for $i \neq j$ $(\perp)$

② $\|b_i\| = 1$ (unit vectors / normal)

Call basis $\boxed{\text{orthonormal}}$

Term collect n orthonormal vectors for a space of dimension n into matrix Q

$$Q = [q_1, q_2 \ldots q_n]$$

orthogonal matrix

Properties: (1) q_i are a basis

(2) $Q^T Q = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & & \ddots & & \\ & & & & 1 \end{bmatrix} = I$

(3) $Q^{-1} = Q^T$

ex so Q is our orthonormal basis

$$[x]_E = Q\,[x]_Q$$

$$[x]_Q = Q^T\,[x]_E$$

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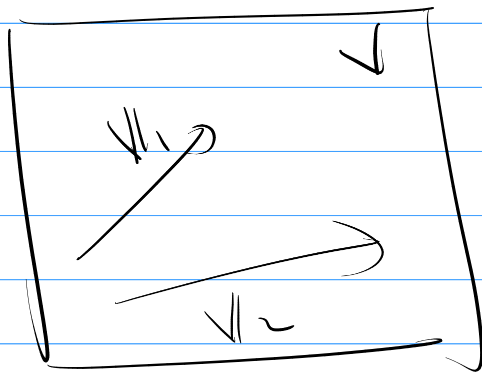
reasons why $\boxed{\text{orthonormal}}$ is better

#1 $x \to [x]_{Basis}$ ← coord. $\quad B = [b_1, b_2 \ldots b_k]$

if B is a set of orthonormal basis

$$[x]_B = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}_B = \begin{bmatrix} \langle x, b_1 \rangle \\ \langle x, b_2 \rangle \\ \vdots \\ \langle x, b_k \rangle \end{bmatrix}_B \qquad c_i = \langle x, b_i \rangle$$

Ans) For any Vector Space with $\langle , \rangle$ defined and orthonormal basis B

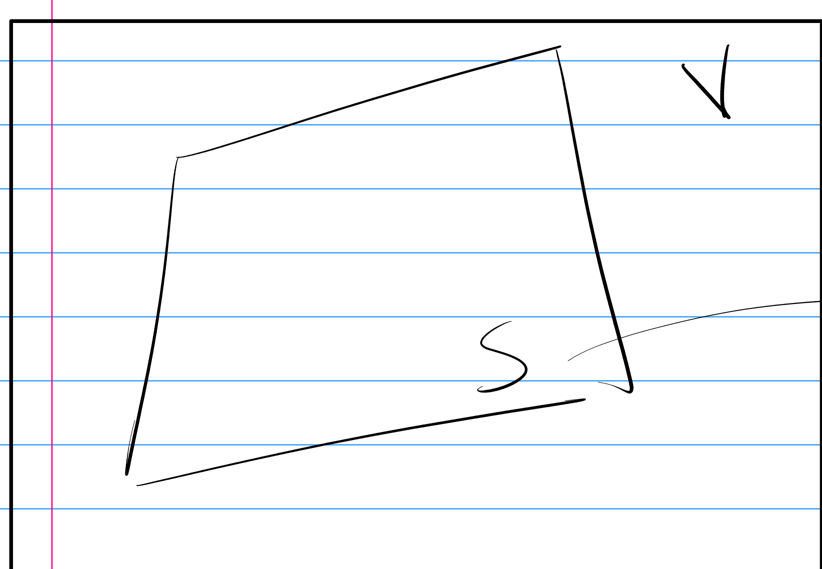


Find $[v_1]_B = a_1$ coord

$[v_2]_B = a_2$ coord

Fun Facts: (2) $\langle v_1, v_2 \rangle = \boxed{a_1^T a_2}$

(3) $\|v_1\|_2 = (\langle v_1, v_1 \rangle)^{1/2} = \boxed{(a_1^T a_1)^{1/2}}$



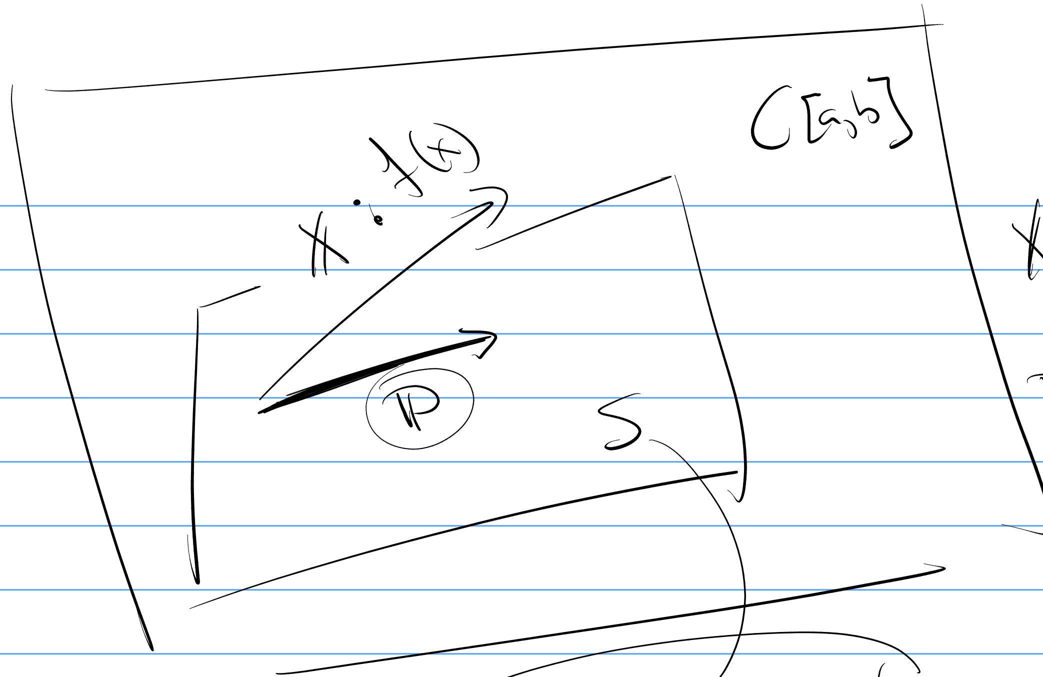
S is a subspace made from an orthonormal set...

$S = \text{Span}(\underbrace{u_1, u_2, \dots, u_k}_{\text{orthonormal basis for } S \text{, a subspace}})$

Why?: (1) $\dim(C[a,b]) = \infty$

(2) $\dim(P) = \infty$

So



$X \notin \mathbb{P}$

$\mathbb{P}$ is the "best" approximate

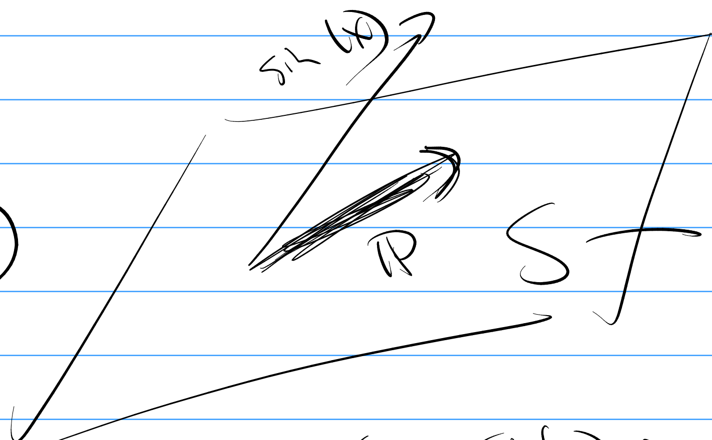
$S = \text{Span}(\underline{u_1}, \underline{u_2}, \ldots, \underline{u_k})$

orthonormal

$$[\mathbb{P}]_u = \begin{bmatrix} \langle X, u_1 \rangle \\ \langle X, u_2 \rangle \\ \vdots \\ \langle X, u_k \rangle \end{bmatrix}_u = \underline{c_1} u_1 + \underline{c_2} u_2 + \ldots \underline{c_k} u_k$$

application:

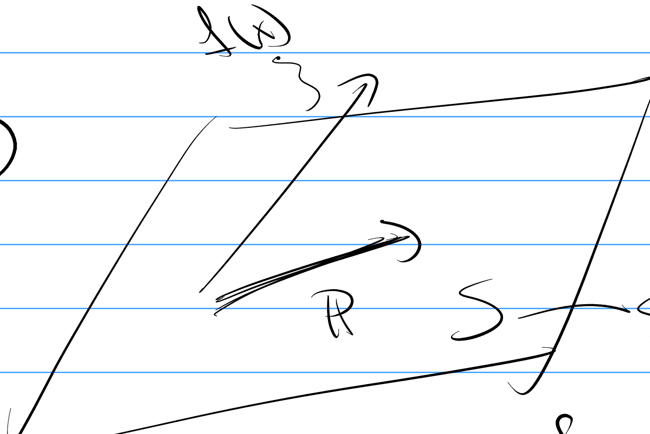
#1



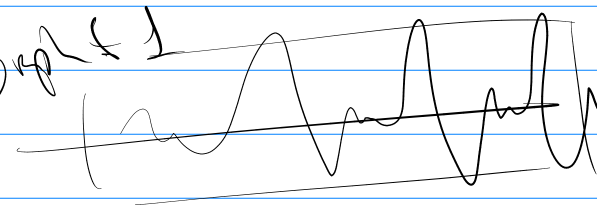
Span(polynomials)

So $\sin(x) \approx$ some "good" polynomial

#2



graph of f



$\text{Span}(\sin(x), \cos(x), \sin(2x), \cos(2x), \ldots, \sin(\infty x), \cos(\infty x))$

Fourier transform