Math 112

Today: 3.2/3.3

Friday: 3.4

Monday: No School

Tues: Review

Wed: Exam 1 (ch1 to ch3)

Q's 3.1 #1, #4

3.1 #1 $f(x) = 7\left(x - \frac{1}{3}\right)^3 (x+2)\left(x + \frac{7}{3}\right)^2$

zero: $x = \frac{1}{3}$ multiplicity is 3

$x = -2$ " " 1

$x = -\frac{7}{3}$ " " 2

lead term x^3 lead term x $(-x^3)$ lead term

#4 $f(x) = 7\boxed{\left(\cancel{x} - \frac{1}{3}\right)^3}\boxed{(x+2)}\boxed{\left(\frac{7}{3} - x\right)^3}$

mult out?

$\left(x - \frac{1}{3}\right)^3 = \left(\boxed{x^3} + x^2 + \frac{1}{3}x - \frac{1}{27}\right)$

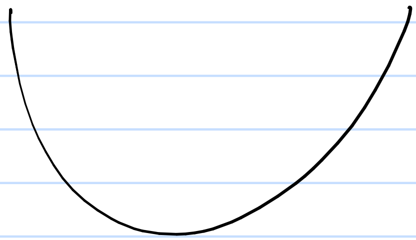
$(\square - \Delta)^3 = \square^3 - 3\square^2\Delta + 3\square\Delta^2 - \Delta^3$

$f(x) = -7x^7 + \dots$

Basic "Shapes" of poly.

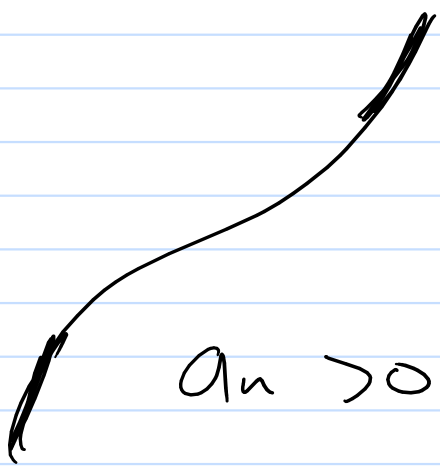
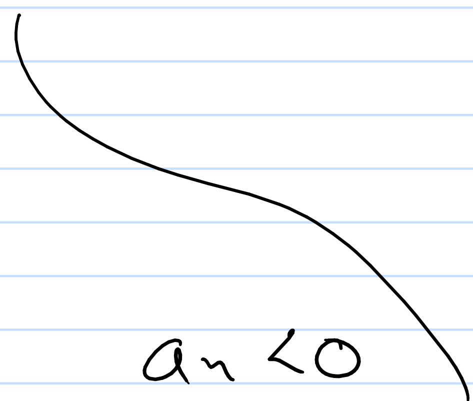
based on __lead term__ $\underline{a_n} x^n$

n is even

$a_n > 0$ $a_n < 0$

n is odd

$a_n > 0$ $a_n < 0$

Long Division

$$\frac{\text{poly}_1}{\text{poly}_2} \qquad \text{poly}_1 \div \text{poly}_2$$

$$\begin{array}{r|l} & \boxed{\text{quotient}} \\ \hline \text{poly}_2 & \text{poly}_1 \end{array}$$

must write $\boxed{\text{all}}$ terms

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R(x)

$$\frac{x^4-1}{x^2-1} \rightarrow$$

$$\begin{array}{r|l} & \quad x^2 \qquad\qquad\quad +1 \\ \hline x^2+0x-1 & x^4+0x^3+0x^2+0x-1 \\ & -(x^4+0x^3-x^2) \\ \hline & \qquad\qquad x^2+0x-1 \\ & \qquad\quad -(x^2+0x-1) \\ \hline & \qquad\qquad\qquad\quad 0 \end{array}$$

So $\dfrac{x^4-1}{x^2-1} = x^2+1$

or $(x^4-1) = (x^2-1)(x^2+1)$

$$\frac{x^3+3x^2-2}{x^2+1} \rightarrow$$

$$\begin{array}{r|l} & \qquad\qquad x+3 \\ \hline x^2+0x+1 & x^3+3x^2+0x-2 \\ & -(x^3+0x^2+x) \\ \hline & \qquad 3x^2-x-2 \\ & \quad -(3x^2+0x+3) \\ \hline & \qquad\qquad -x-5 \end{array}$$

Says

$$\boxed{\frac{x^3+3x^2-2}{x^2+1} = (x+3) + \frac{-x-5}{x^2+1}}$$

Synthetic is [only] for $(x-c)$ divides $p(x)$

$\dfrac{p(x)}{x-c} \rightarrow$ c | coef of $p(x)$ [all of them]

add

0

mult to get

$$\boxed{\frac{4x^5 - 2x^3 + x}{x+3}}$$

$x - (-3)$

long division $\rightarrow$ $x+3 \overline{)\, 4x^5 + 0x^4 - 2x^3 + 0x^2 + x + 0}$

syn. division

$$\begin{array}{r|rrrrr|r} -3 & 4 & 0 & -2 & 0 & 1 & 0 \\ & 1 & -12 & 36 & -102 & 306 & -921 \\ \hline & 4 & -12 & 34 & -102 & 307 & -921 \\ & x^4 & x^3 & x^2 & x & c & r \end{array}$$

$$\frac{4x^5 - 2x^3 + x}{x+3} = \left(4x^4 - 12x^3 + 34x^2 - 102x + 307\right) + \frac{(-921)}{x+3}$$

Fact & Syn. division

divide by $x - c$

remainder is $p(c)$

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factoring $p(x)$ w/ $(x - \text{roots})$

$$p(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$$

$$= a_n (x - r_1)^{m_1} (x - r_2)^{m_2} \cdots$$

$\uparrow$ root multiplicity

(ex) your problem has roots of $x = 3$ w/ mult. 1

$x = -2$ w/ mult. 3

$x = 1/2$ w/ mult. 2

$$p(x) = a_n (x-3)^1 (x+2)^3 (x - 1/2)^2$$

ex roots $x = 1$ w/ mult. 2
$x = -3$ w/ mult. 2
$x = 2$ w/ mult. 1

and it goes through $(0, 4)$

$$p(x) = a_n (x-1)^2 (x+3)^2 (x-2)^1$$

use $(0, 4)$
$x=0$, $y=4$

$$4 = a_n \underbrace{(-1)^2 (3)^2 (-2)^1}$$

$$4 = -18 a_n$$

$$a_n = -\frac{2}{9}$$

$$\boxed{p(x) = -\frac{2}{9} (x-1)^2 (x+3)^2 (x-2)}$$

Finding Roots

$$p(x) = a_n x^{n} + \cdots + a_2 x^2 + a_1 x + a_0$$

n – degree

thm: how many roots? n

finding possible roots to check?

1st: if $a_0, a_1, \ldots, a_n$ are all integers?

try $\dfrac{\text{factors of } a_0}{\text{factors of } a_n}$ (use syn. division) try $\underline{\underline{\text{all}}}$ $\dfrac{\text{factors of } a_0}{\text{factors of } a_n}$

c | quotient | r

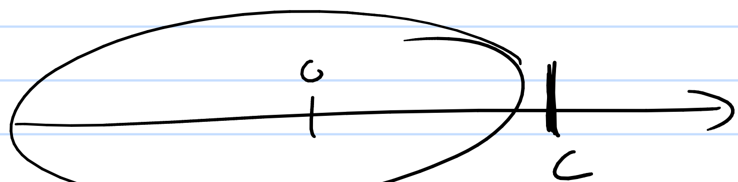
① if $r = 0 \to$ we found a root (also found factors)

$(x-c)(\text{quotient})$

② $\boxed{\underline{\underline{r \neq 0}}\,?!}$

(a) $c(>0)$ | r

you see everyone has same sign

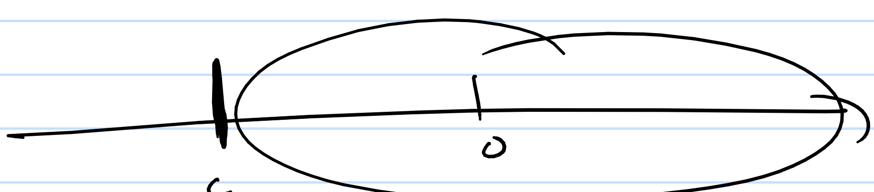


Roots have to be to the left of c

(b) $c(<0)$ | r

$\underline{\underline{\text{alternating signs}}}$

then



Roots have to be to the right of c