Math 112

$\boxed{Q's}$ 3.2 5, 6, 7, 10

$$\frac{8x^4+2x^3+4x^2-5x-9}{x+9}$$

$$\frac{P(x)}{x-c}$$

c | coef of P(x)

add

mult

5 Use synthetic division to find the quotient and remainder when $f(x) = 8x^4 + 2x^3 + 4x^2 - 5x - 9$ is divided by $g(x) = x + 9$.

The quotient is ______.

The remainder is ______.

6 $c = 3$ is a zero of $P(x) = x^3 - 14x^2 + 61x - 84$. Find all other zeros of $P(x)$.

$x_1 =$ ______ and $x_2 =$ ______ with $x_1 < x_2$.

7 Is $(x - 6)$ a factor of $f(x) = -7x^4 - 2x^3 + 6x^2 + 3x + 5$?

Answer yes or no: ______

(5)

-9	8	2	4	-5	-9
		-72	630	-5706	51399
	8	-70	634	-5711	51390
	x^3	x^2	x	c	

$$q(x) = 8x^3 - 70x^2 + 634x - 5711$$

$$r = 51,390$$

$$\frac{P(x)}{x+9} = q(x) + \frac{r}{x+9}$$

or $p(x) = (x+9)(q(x)) + r$

6. $c = 3$ is a zero of $P(x) = x^3 - 14x^2 + 61x - 84$. Find all other zeros of $P(x)$.

$x_1 =$ 4 and $x_2 =$ 7 with $x_1 < x_2$.

7. Is $(x-6)$ a factor of $f(x) = -7x^4 - 2x^3 + 6x^2 + 3x + 5$?

Answer yes or no:

Zero / root

$x = c$ is zero/root

Means $(x-c)$ is a factor

$$\begin{array}{r|rrrr} 3 & 1 & -14 & 61 & -84 \\ & & 3 & -33 & 84 \\ \hline & 1 & -11 & 28 & 0 \end{array}$$

$$\begin{array}{r|rrr} 4 & 1 & -11 & 28 \\ & & 4 & -28 \\ \hline & 1 & -7 & 0 \end{array}$$

x c r

$x^3 - 14x^2 + 61x - 84 = (x-3)(x^2 - 11x + 28)$

$x = 4$ is a zero

$= \boxed{(x-3)(x-4)(x-7)}$

$x = 3$ | $\boxed{x=4}$ | $\boxed{x=7}$

Zeros are factors

$-4, 1, 6$ are zeros

$P(x) = a_n (x+4)\boxed{(x-1)(x-6)}$

$= a_n (x+4)(x^2 - 7x + 6)$

$= a_n (x^3 - 3x^2 - 22x + 24)$

$= a_n x^3 \boxed{-3a_n} x^2 - 22a_n x + 24a_n$

$-6 = -3a_n$

$2 = a_n$

so $a_n = 2$ $\quad -3a_n = -6$

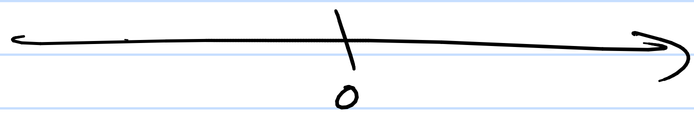
$= 2(x^3 - 3x^2 - 22x + 24) = \boxed{2x^3 - 6x^2 - 44x + 48}$

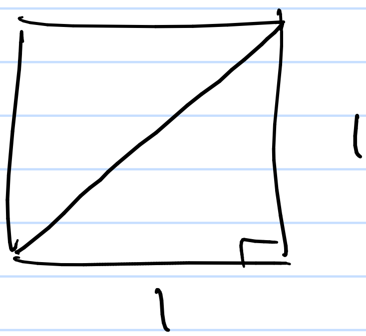
$p(x)$ is of degree n

the $p(x)$ has n roots that are real and/or complex

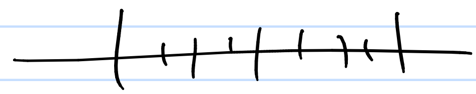
Complex?

Reals:





$$\frac{a}{b} = a\left(\frac{1}{b}\right)$$



$$x^2 = 4$$

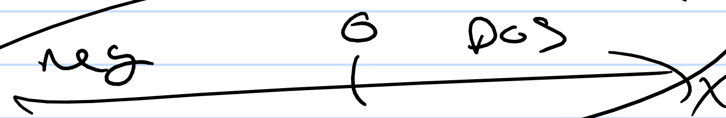
$$x^2 - 4 = 0$$

$$(x+2)(x-2) = 0$$

$$x = -2 \quad x = 2$$

$$x^2 = -1$$

$$x^2 + 1 = 0$$



$x = i$ such that $i^2 = -1$

$i = \sqrt{-1}$ imaginary unit

$i = \sqrt{-1}$

$i^2 = -1$

$i^3 = (i^2)(i) = -i$

$(i^4) = (i^2)(i^2) = (-1)(-1) = 1$

$i^5 = i^4 \cdot i = (1)\, i = i$

$$i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1 \quad i^5 = i \quad i^6 = -1 \quad i^7 = -i \quad i^8 = 1 \quad \vdots$$

$$i^{105} = i^{100} \cdot i^5 = \underbrace{(i^4)^{25}}_{=1} i^5 = i$$

Complex number $\qquad \underbrace{a}_{\text{real}} + \underbrace{b}_{\text{imaginary}} i$

$\pi = \pi + 0i$

$3i = 0 + 3i$

$1 + 2i$, $\sqrt{4} - \frac{1}{3}i$, etc

$(1 + 3i) + (-2 - i) = \boxed{-1 + 2i}$

$(1 + 3i)(-2 - i) = -2 - i - 6i - 3i^2$

$= \boxed{1 - 7i}$

Conjugates $(1+2i)(1-2i)$

$= 1 - 4i^2 = 1 + 4 = \boxed{5}$

$$\frac{(1+2i)(1+3i)}{(1-3i)(1+3i)} = \frac{1+5i+6i^2}{10}$$

$$= \frac{-5+5i}{10} = \boxed{-\frac{1}{2} + \frac{1}{2}i}$$

roots for polynomials w/ real coeff.

if you have complex roots

they are complex conj. pairs

they come from irreducible quadratics

$P = (x-2)^3 (x+1)^2 (x^2+2x+7)$

Descartes' Rule of Signs

$$p(x) \qquad \underbrace{3x^2}_{\text{change}} - \underbrace{2x}_{\text{change}} + 1$$

two pos real solutions or

(subtract by 2)

$p(-x)$