Math 112

Q's

A degree 4 polynomial $P(x)$ with integer coefficients has zeros $5i$ and 1, with 1 being a zero of multiplicity 2. Moreover, the coefficient of x^4 is 1. Find the polynomial.

$P(x) =$

$0 + 5i$
$0 - 5i$] conj. pairs

$$p(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad \text{polynomial}$$

or

$$p(x) = a_4 (x - r_1)(x - r_2)(x - r_3)(x - r_4) \quad \text{factored version}$$

roots!

$$= a_4 (x-1)(x-1)(x-5i)(x+5i)$$

$$= a_4 (x^2 - 2x + 1)(x^2 + 25)$$

$$= a_4 [x^4 - 2x^3 + 26x^2 - 50x + 25]$$

$$= a_4 x^4 - 2a_4 x^3 + 26 a_4 x^2 - 50 a_4 x + 25 a_4$$

coef is 1
so $a_4 = 1$

$$\boxed{p(x) = x^4 - 2x^3 + 26x^2 - 50x + 25}$$

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play a bit

$$p(x) = x^4 - 2x^3 + 26x^2 - 50x + 25$$

told that $x = 5i$ is a root

Now factor! (using long division)

play a bit

$$p(x) = x^4 - 2x^3 + 26x^2 - 50x + 25$$

told that $x = 5i$ is a root

Now factor! (using long division)

how? $\boxed{\text{root is factor}}$ $x = c$ is a root/zero $\Leftrightarrow$ $(x-c)$ is a factor

$x = 5i$ is root then $(x-5i)$ is a factor

we also know (complex conj. pairs) $\rightarrow$ $x = -5i$ is root then $(x+5i)$ is a factor

mult.

or together (x^2+25) is a factor.

$$\begin{array}{r}
x^2 - 2x + 1 \\
x^2 + 0x + 25 \,\big)\, \overline{x^4 - 2x^3 + 26x^2 - 50x + 25} \\
\underline{x^4 + 0x^3 + 25x^2} \\
-2x^3 + x^2 - 50x \\
\underline{-2x^3 + 0x^2 - 50x} \\
x^2 + 0x + 25 \\
\underline{x^2 + 0x + 25} \\
0
\end{array}$$

$$(x^4 - 2x^3 + 26x^2 - 50x + 25) = (x^2+25)(x^2-2x+1)$$

$$= \boxed{(x^2+25)(x-1)(x-1)}$$

Complex roots are in the irreducible quadratics

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One solution of the the equation

$$p(x) = x^4 - 9x^3 + 30x^2 - 33x - 13 = 0$$

is $x = 3 - 2i$. The equation also has two real solutions. The smaller is [ ], and the larger is [ ].

Hint: Figure out a suitable quadratic factor and divide p by that factor.

root $= 3-2i$ so another root is $3+2i$

$(x-(3-2i))(x-(3+2i))$ are two factors

$= x^2 - (3+2i)(x) - (3-2i)(x) + (3-2i)(3+2i)$

$= x^2 - 3x - \cancel{2ix} - 3x + \cancel{2ix} + 9 + 4$

$= (x^2 - 6x + 13)$ is a factor

$$\begin{array}{r} x^2 - 3x - 1 \\ x^2-6x+13 \,\overline{)\, x^4 - 9x^3 + 30x^2 - 33x - 13} \\ \text{sub} \quad x^4 - 6x^3 + 13x^2 \\ \hline -3x^3 + 17x^2 - 33x \\ \text{sub} \quad -3x^3 + 18x^2 - 39x \\ \hline -x^2 + 6x - 13 \\ -x^2 + 6x - 13 \\ \hline 0 \end{array}$$

$x^4 - 9x^3 + 30x^2 - 33x - 13 = \underbrace{(x^2-6x+13)}_{\text{roots } 3 \pm 2i} \underbrace{(x^2-3x-1)}_{\text{roots?}}$ (by quad. formula)

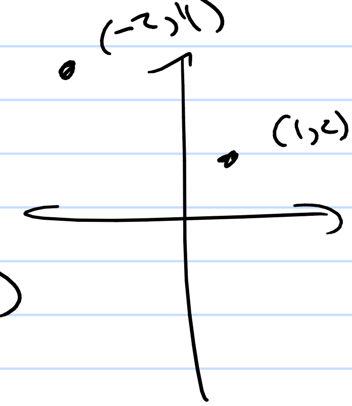
roots of $x^2 - 3x - 1$ are $x = \frac{3}{2(1)} \pm \frac{\sqrt{9+4}}{2(1)}$

$= \frac{3}{2} \pm \frac{1}{2}\sqrt{13}$

Exam 1

12 probs @ 10 pts each
110 pts = 100%

ch 1 (4 probs)



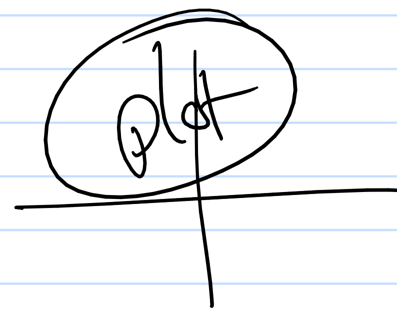
① plot several graphs

Section 1.2

(a) given points (1,2) (-2,4)

(b) given eqn's (ex) $y = x^2 + 1$
(more)

→ make a table of points



② Section 1.4 / 1.5

Function Notation / Arithmetic

(a) (ex) given $f(x) = \dfrac{x^3 - 3x}{2x - 1}$ (Evaluate)

find $f(1)$, $f(x+h)$, $f(2/3)$, etc

$$f(x+h) = \frac{(x+h)^3 - 3(x+h)}{2(x+h) - 1}$$

(b) (ex) $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$

w/ domain question.

③ transformations & graphs | Section 1.6 / 1.7

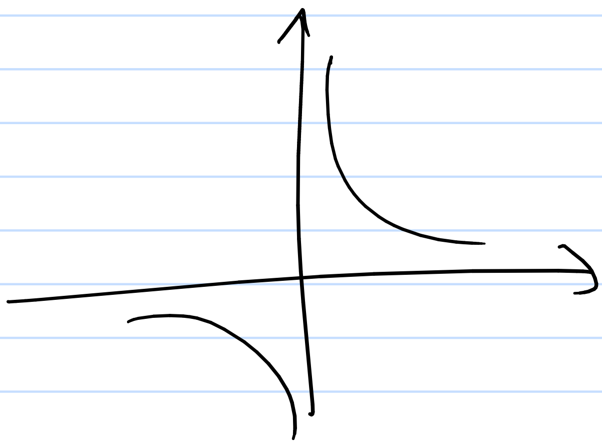
(a) graph some simple function

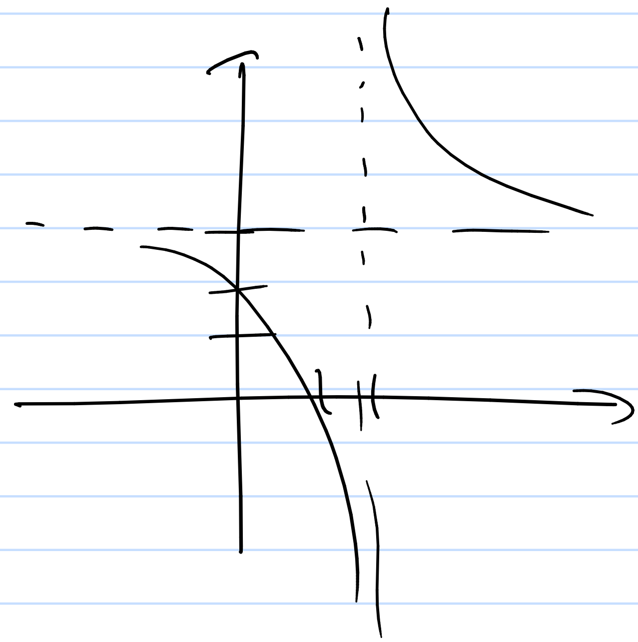
(ex) $f(x) = x$, $f(x) = x^2$, $f(x) = x^3$

$f(x) = \frac{1}{x}$, $f(x) = |x|$

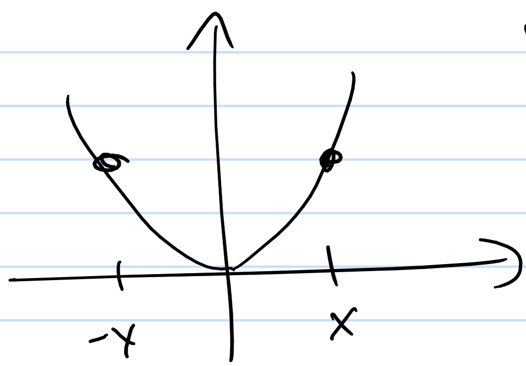
(3) (b) translations

$$f(x) = \frac{1}{x-2} + 3$$

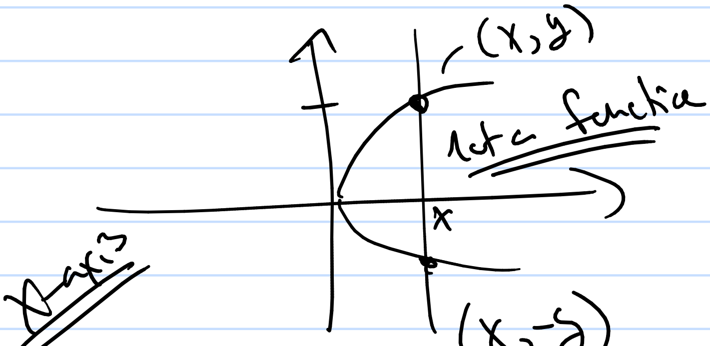


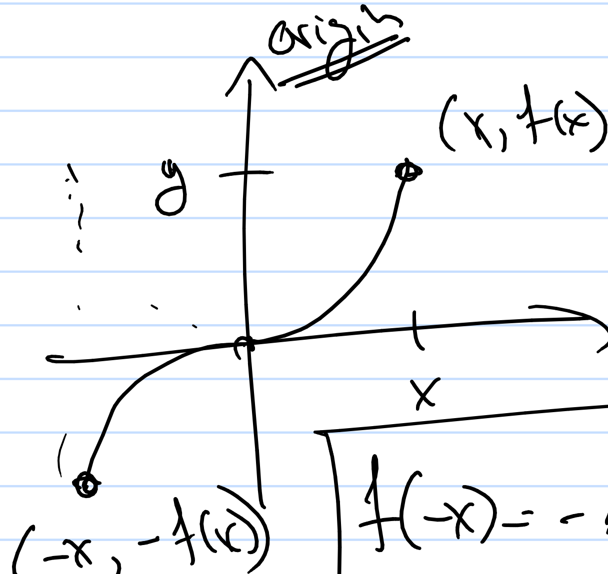


(4) Section 1.6 Symmetry.



$$\boxed{f(-x) = f(x)}$$ y-axis





$$\boxed{f(-x) = -f(x)}$$

(ex) $f(x) = x^3 - x$

$$f(-x) = (-x)^3 - (-x) = -x^3 + x = -f(x)$$

Ch 2 — 4 probs

(1) 2.1 — given points / slopes → write linear eqn.
(could be a word problem)

(2) 2.2 — solve eqn w/ abs. value.

(a) ex $2|x| + 4 = 3$

(b) ex $4|x-3| = |x+2|$

(3) 2.3 — word problem. ex Fencing HW prob.

(4) 2.4 — quad. inequalities / Linear Inequalities

(a) ex $3|x+1| > 1$

(b) ex $x^2 + 2x \geq x + 4$

Ch 3 — 4 probs

ex $f(x) = (x-2)^3(x+1)$

$x=2$ mult. is 3 — $x=-1$ mult. is 1

(1) given $f(x)$ and its factors..
(a) zero?
(b) end point behavior

(2) Factor given zeros using syn. division.

(3) Factor given zeros using long division.

(4) given complex zero → factor using long division.

$$f(x) = 3(x-3)^{12}(2-x)^{11} = -3x^{23} + \cdots$$

Zero: 3 Zero: 2

mult: 12 mult: 11