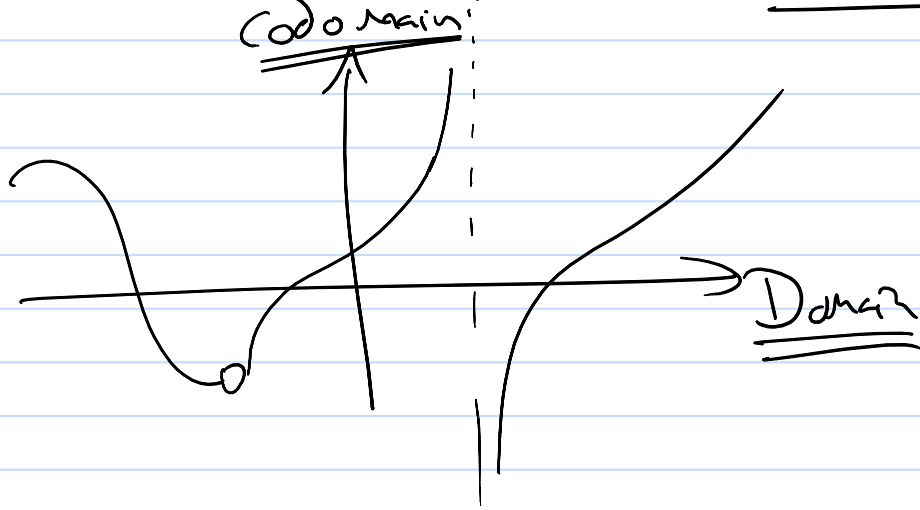
Math 112

Rationals

rational = $\dfrac{\text{polynomial}}{\text{polynomial}}$

last class: add, subtract, mult., divide of rationals

Graphs



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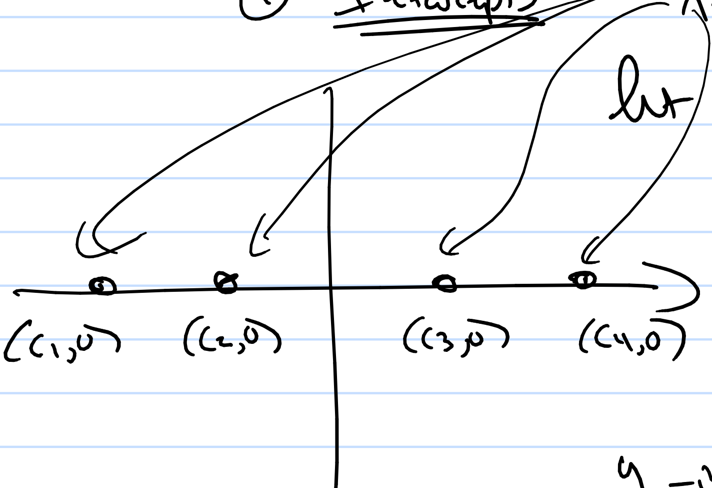
graphical features of rationals ($r(x)$ is a rational function)

① Intercepts — x-intercepts (zeros of a function)

let $y=0$ so $\boxed{0 = r(x)}$

↑ Solve



y-intercepts

let $x=0$ point $(0, r(0))$

(2) Table of Values?

4) Need to know $\boxed{\text{Domain?}}$ ← all reals except we must exclude any numbers that make a denominator $= 0$

5) Special physical features of rationals . .

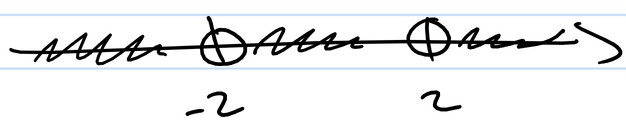
Asymptotes, **Holes in the graph**

"Domain Problems": any numbers that make a denominator $= 0$.

type 1	type 2
Vertical Asymptote	Hole
(non-removable discontinuity)	(removable discontinuity)

(ex) $r(x) = \dfrac{x^2-x-6}{x^2-4} = \dfrac{(x+2)(x-3)}{(x+2)(x-2)}$

Domain: All reals except $x \neq 2$ $x \neq -2$

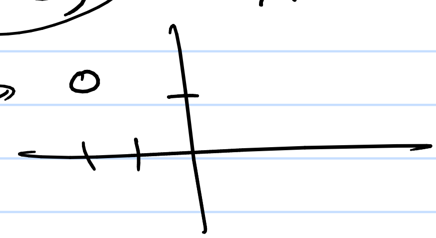


$(-\infty,-2) \cup (-2,2) \cup (2,+\infty)$

$r(x) = \dfrac{\cancel{(x+2)}(x-3)}{\cancel{(x+2)}(x-2)} = \dfrac{x-3}{x-2}$, $x \neq -2$

x	y
-2	$5/4$

(but $x \neq -2$) so Hole!



$f(x) = \frac{x-3}{x-2}$, $x \neq -2$

x	y
-2	5/4
-1	4/3
0	3/2
1	2
3/2	3
1.9	11
$\boxed{2}$	$\boxed{\text{dne}}$
2.1	-9
3	0

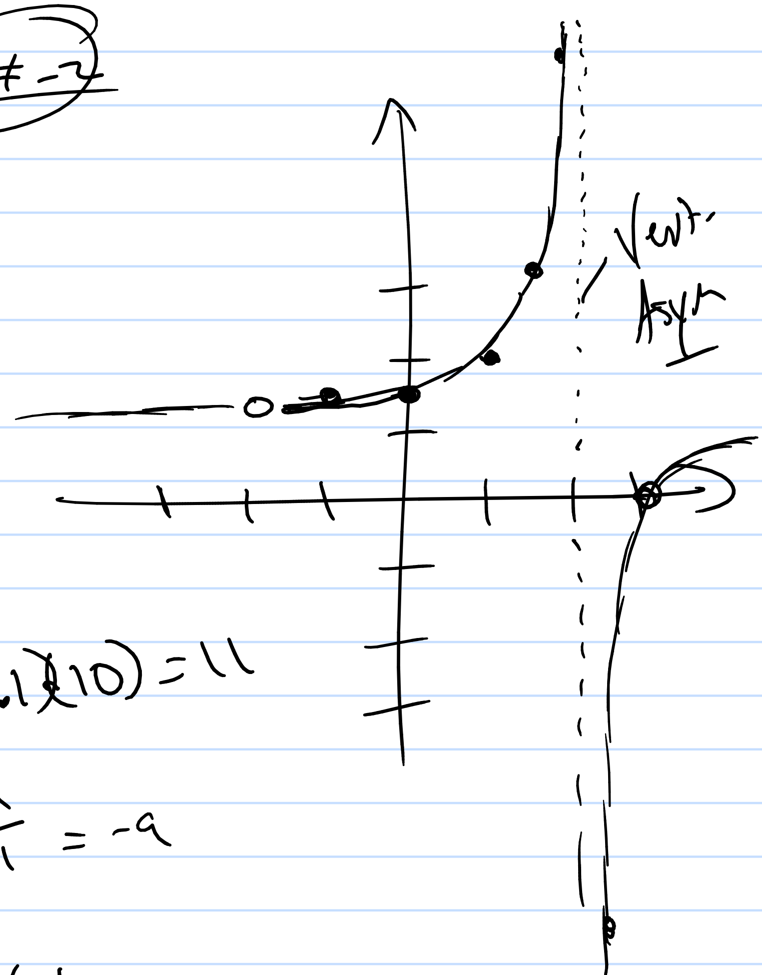
(Hole)

$\frac{3/2-3}{3/2-2} = \frac{-3/2}{-1/2} = \frac{3}{2}\cdot\frac{2}{1} = 3$

$\frac{1.9-3}{1.9-2} = \frac{-1.1}{-.1}$

$= \frac{1.1}{\frac{1}{10}} = (1.1)(10) = 11$

$\frac{2.1-3}{2.1-2} = \frac{-.9}{.1} = -9$

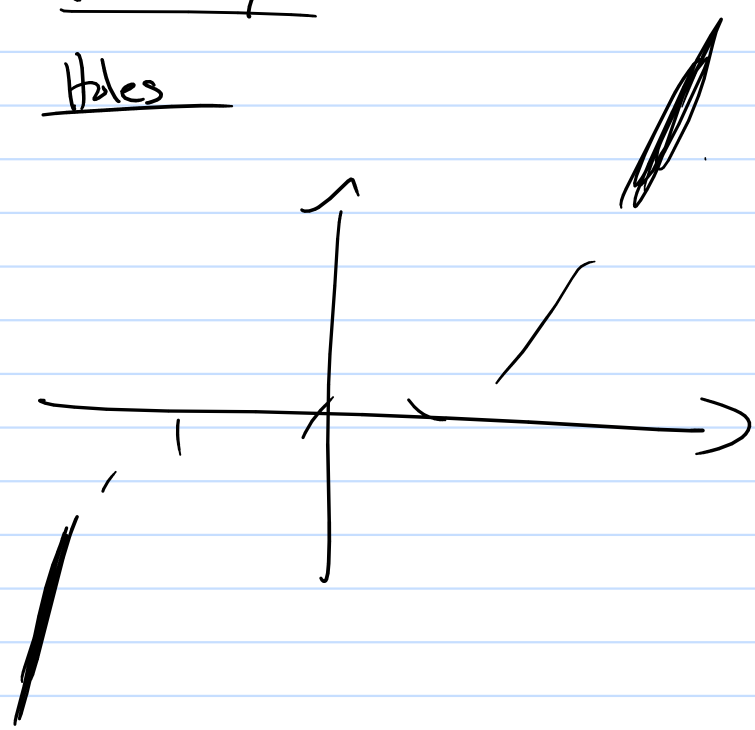


So Domain problems → Vert. asym
→ Holes

$\boxed{\text{End Point behavior?}}$

ex $\boxed{3x^3}-1$

of poly



Rationals?

End point behavior of Rationals

$$r(x) = \frac{p(x)}{d(x)}$$

two polynomials

① if degree of $d(x)$ > degree $p(x)$

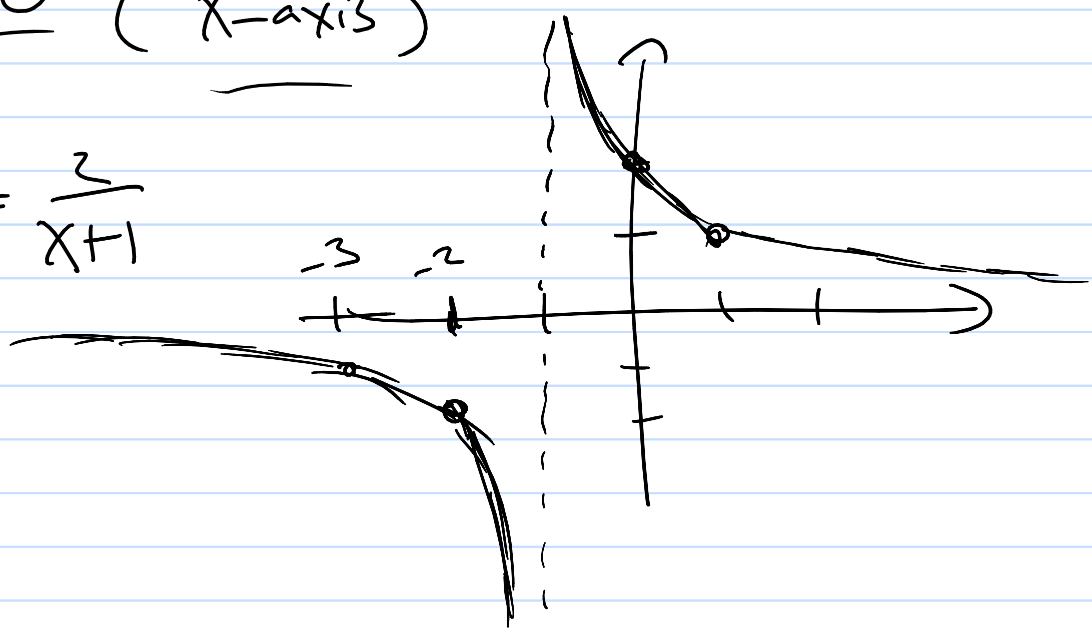
ex $r(x) = \dfrac{3x^2+x-1}{x^3-2}$ or $r(x) = \dfrac{x+2}{x^2-1}$

or $\boxed{r(x) = \dfrac{2}{x+1}}$

then we have a horizontal asymptote of

$y=0$ (x-axis)

→ $r(x) = \dfrac{2}{x+1}$



② degree $d(x)$ = degree $p(x)$

ex $r(x) = \dfrac{3x^3-1}{-4x^3+x^2+1}$

lead coef of $p(x) = 3$

lead coef of $d(x) = -4$

→ horizontal asymptote at $y = \dfrac{\text{lead coef. of } p(x)}{\text{lead coef. of } d(x)}$

so → $y = -\frac{3}{4}$ is the horz. asym

③ degree $d(x)$ < degree $p(x)$

now we use long division ...

$$d(x)\overline{\smash{)}\,p(x)}\quad q(x)$$

$\quad\vdots$

$\underline{\qquad}$

$r(x)$

$\rightarrow$ gives $p(x) = d(x)\,q(x) + r(x)$

or $r(x) = \dfrac{p(x)}{d(x)} = \boxed{\underline{q(x)} + \dfrac{r(x)}{d(x)}}$

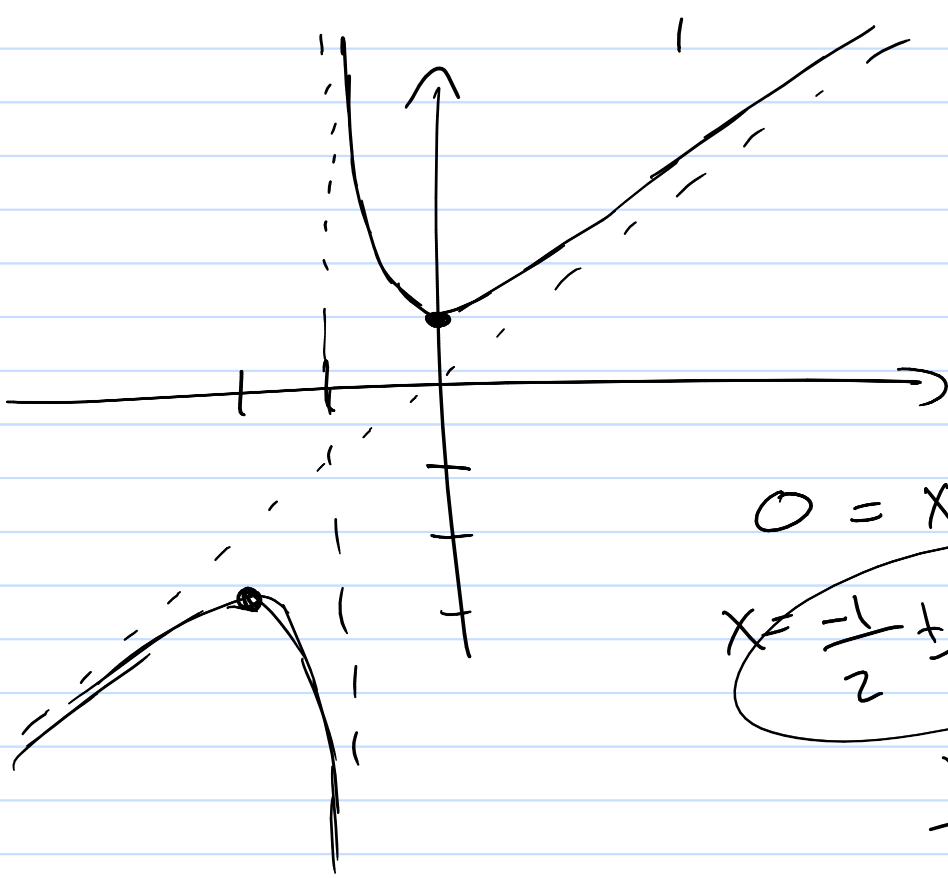
ex $r(x) = \dfrac{x^2+x+1}{x+1}$

$$x+1\overline{\smash{)}\,x^2+x+1}\quad x$$

$\underline{x^2+x}$

so $\boxed{\dfrac{x^2+x+1}{x+1}} = \boxed{x} + \dfrac{1}{x+1}$

asymptote



x	y
0	1
-2	-3

$0 = x^2+x+1$

$x = \dfrac{-1}{2} \pm \dfrac{\sqrt{-3}}{2}$

no x-int.