Math 112

Q's

4.1 #3

Rational functions

$$f(x) = \frac{(3x-1)(x+5)}{(-x-9)(5x-1)}$$ ≡ $$f(x) = \frac{3x^2 + 14x - 5}{-5x^2 - 44x + 9}$$

↑ Factor

used for vertical asym, holes, x-intercepts, domain

rational = polynomial / polynomial

used for horz. asym., slant asym

from domain = 0 - domain: all reals except $x \neq 1/5$ $x \neq 9$

- Vertical asym. @ $x = 1/5$, $x = 9$
- no holes

x-int: (zero & numerator) $x = 1/3$ $x = -5$

$(1/3, 0)$ $(-5, 0)$

b/c degree of numerator = degree of denominator

→ horz. asymptote of $y = \frac{\text{lead coef of numer.}}{\text{lead coef of denom.}}$

So $y = -\frac{3}{5}$

Example 4.2.1. Sketch a detailed graph of $f(x) = \dfrac{3x}{x^2-4}$.

$= \dfrac{3x}{(x+2)(x-2)}$

Solution. We follow the six step procedure outlined above.

Domain: all reals but $x \neq -2$ $x \neq 2$ ←

←

-2 $\quad$ 0 $\quad$ 2

$(-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$ ←

Vert. asym: @ $x = -2$ $x = 2$

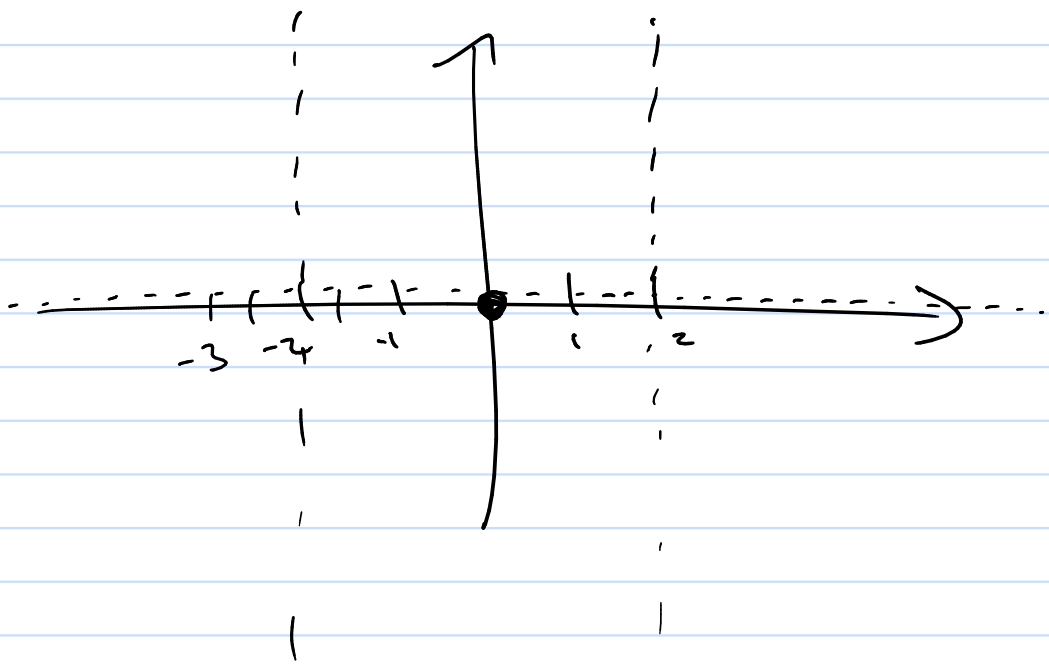
Horz. asym: $y = 0$ b/c degree(num) > deg numer

Intercepts: $(0,0)$ y-int let $x=0 \to y = \dfrac{3 \cdot 0}{(0+2)(0-2)} = 0$

x-int (numerator $=0$) $3x = 0$

$x = 0$

x	y
-3	
-1	
1	
3	



Rational Equations

$$\frac{x+9}{x+1} = -2 \qquad x \neq -1$$

multiply by common denominator

$$(x+1)\left(\frac{x+9}{x+1}\right) = \left(\frac{-2}{1}\right)(x+1)$$

$$x+9 = -2(x+1) \qquad x \neq -1$$

$$x+9 = -2x-2$$

$$x+2x = -2-9$$

$$3x = -11$$

$$x = -11/3$$

Rational Inequalities

$$\frac{x+9}{x+1} < -2$$

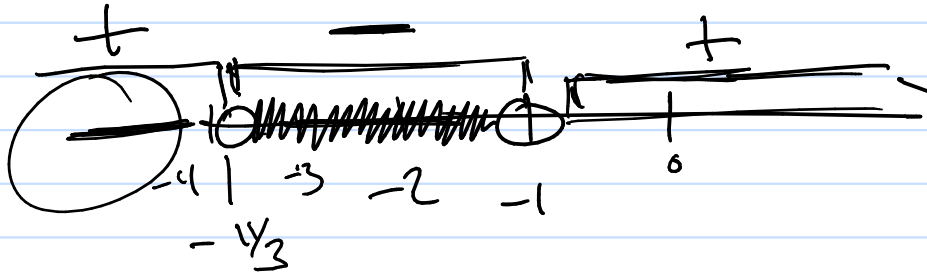
$$\frac{x+9}{x+1} + 2 < 0$$

Need $\boxed{\frac{p(x)}{d(x)} < 0}$

$$\frac{x+9}{x+1} + \frac{2(x+1)}{(x+1)} < 0$$

$$\frac{(x+9)+2(x+1)}{x+1} < 0 \rightarrow \boxed{\frac{3x+11}{x+1} < 0}$$

Sign diagram



$$\boxed{-11/3 < x < -1}$$

$$\boxed{(-11/3, -1)}$$

Rational Inequalities

$$\frac{x+9}{x+1} \geq -2$$

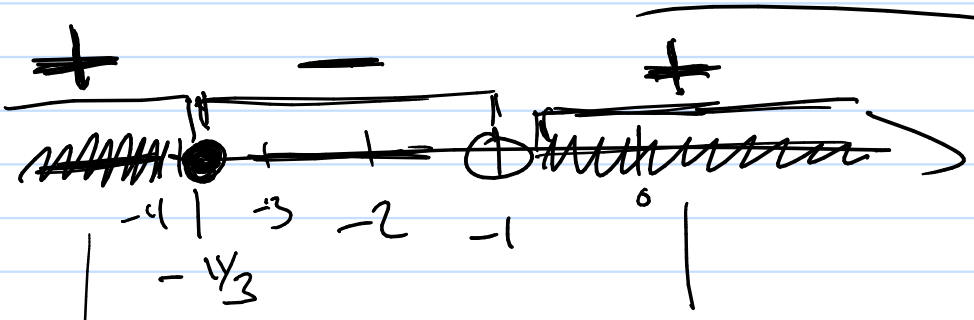
Need $\frac{p(x)}{q(x)} < 0$

$$\frac{x+9}{x+1} + 2 \geq 0$$

$$\frac{x+9}{x+1} + 2\frac{(x+1)}{(x+1)} \geq 0$$

$$\frac{(x+9) + 2(x+1)}{x+1} < 0 \rightarrow \boxed{\frac{3x+11}{x+1} \geq 0}$$

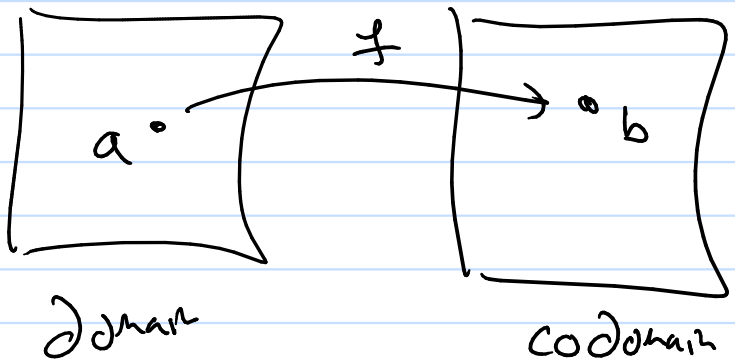
Sign diagram



$$x \leq -11/3 \quad \cup \quad x > -1$$

$$(-\infty, -11/3] \cup (-1, +\infty)$$

Funciones $f(x), g(x)$

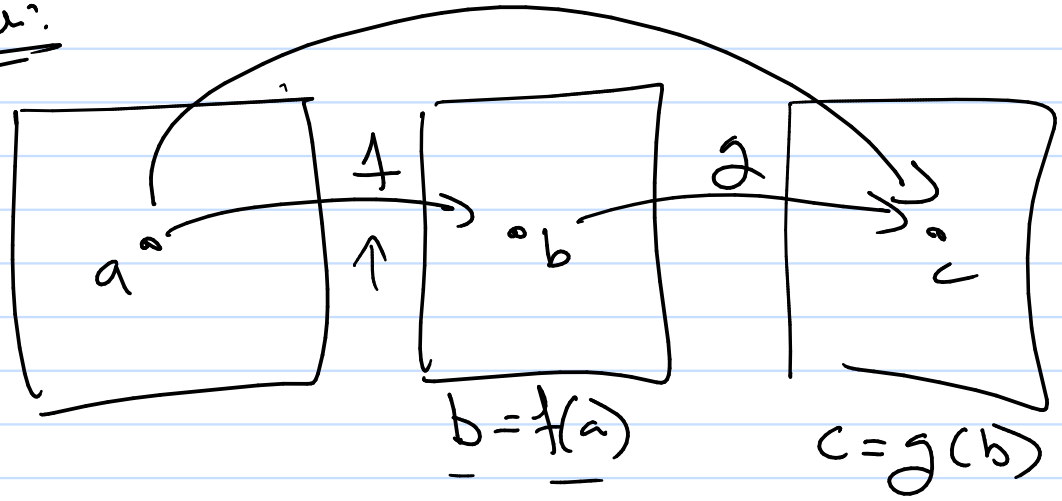


$b = f(a)$

Operaciones

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)\,g(x)$$
$$(f/g)(x) = f(x)/g(x)$$

Composición:



$$g(b) = g(f(a)) = c$$
$$= (g \circ f)(a) = c$$

Def: Composición $(g \circ f)(x) = g(f(x))$

$f(x) = x^2 + 2x - 1$ $\quad$ $g(x) = \sqrt{3x - 4}$

$f(\square) = \square^2 + 2\square - 1$

$g(\square) = \sqrt{3\square - 4}$

$(g \circ f)(x) = g(f(x))$

$= g(x^2 + 2x - 1)$

$= \sqrt{3(x^2 + 2x - 1) - 4}$

$=$ simplify ... $=$

Domain?

Domain check now!

$3(x^2 + 2x - 1) - 4 \geq 0$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{3x - 4})$

$= (\sqrt{3x-4})^2 + 2\sqrt{3x-4} - 1$

$= ?$ Simplify