Math 112

Friday is a recorded lecture (no in-person)

Q's

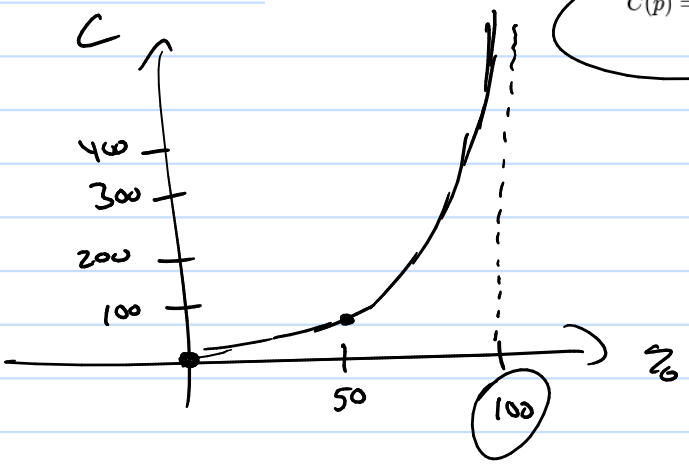
4.1 #7

$0 - 100$

The cost in dollars for removing p percent of pollutants from a river in Smith County is

$$C(p) = \frac{80000p}{100 - p}.$$

→ Vert. asym @ $p=100$



p	C
0	$0
50	$80,000
99	$7,920,000

Composition: $f(x) = x^2 + 2x$ $\quad g(x) = 3x + 2$

$f(\square) = \square^2 + 2\square$ $\quad g(\square) = 3\square + 2$

$(f \circ g)(x) = f(g(x)) = f(3x+2)$

$= (3x+2)^2 + 2(3x+2)$

$= 9x^2 + 12x + 4 + 6x + 4$

$= \boxed{9x^2 + 18x + 8}$

$(g \circ f)(x) = g(f(x)) = g(x^2 + 2x) = 3(x^2 + 2x) + 2$

$= \boxed{3x^2 + 6x + 2}$

Given operations on objects we like to know two things

① How to do nothing (Identity)

② How to undo an operation (Inverse)

(ex) real numbers and addition.

$$3\pi + 0 = 3\pi$$

↖ called the additive identity

$$x + 4 + (-4) = x + 0 = x$$

$$4 + (-4) = 0$$

↑ 4's additive inverse

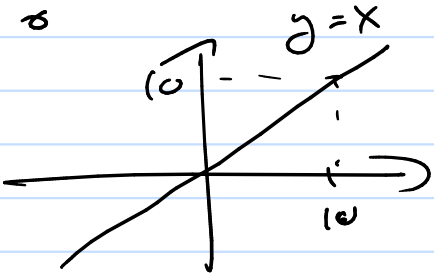
What about composition of functions?

$f \circ g$

Identity? $I(x) = x$

$$(f \circ I)(x) = f(x)$$

$$f(I(x)) = f(x)$$



$I(x) = x$ $\qquad (I \circ f)(x) = I(f(x)) = f(x)$

$I(\square) = \square$

Identity function under composition.

Inverse? under composition?

Note:

Additive Inverse of x

$x + (-x) = 0$

Multiplicative Inverse of x

$x \cdot (x^{-1}) = 1$

Notice: $x \neq 0$ has mult. inv.

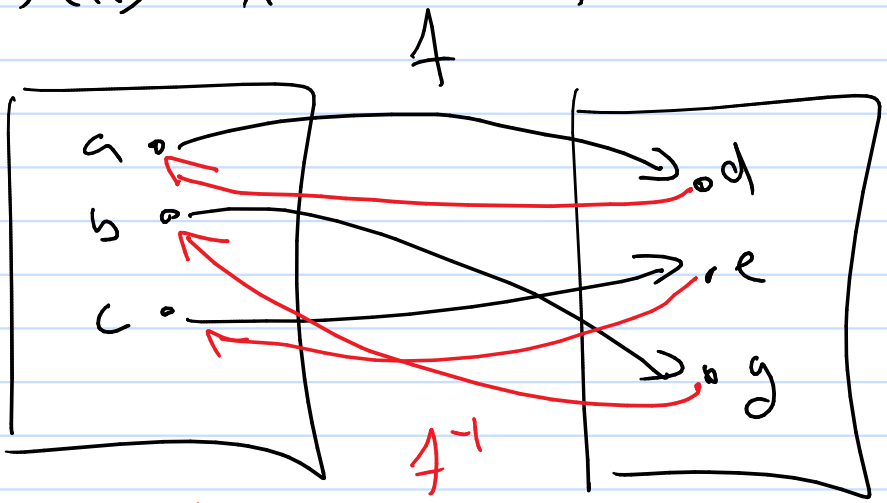
$(f \circ \square)(x) = x$

a function label f's inverse function under composition to be $f^{-1}(x)$

$(f \circ f^{-1})(x) = x$

$(f^{-1} \circ f)(x) = x$

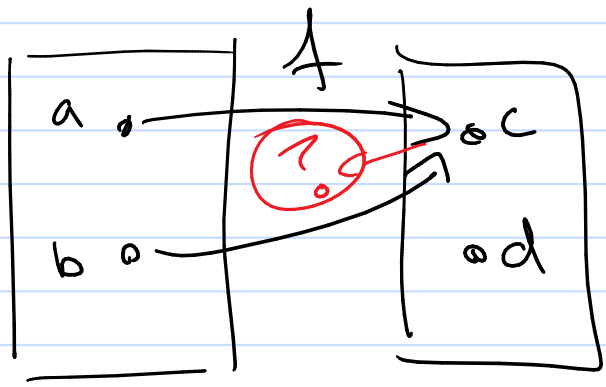
$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$



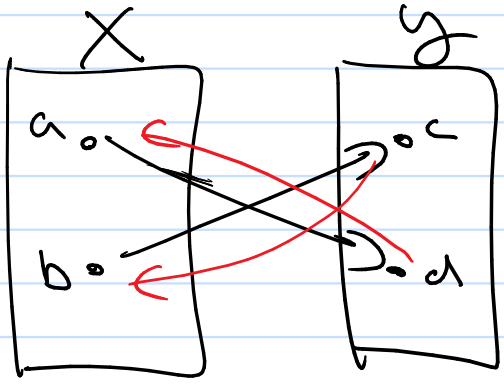
$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$

Finding inverses?

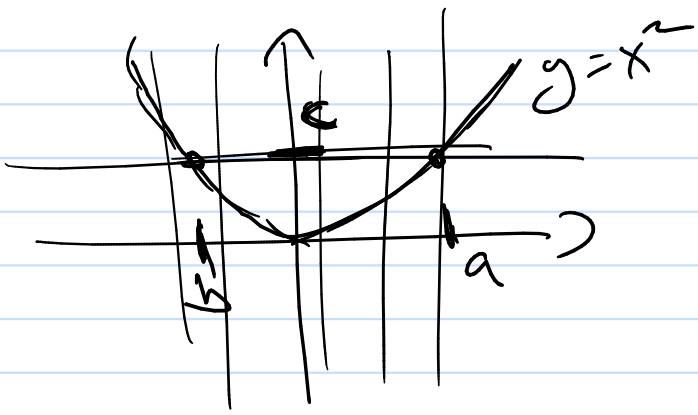
1st Does f even have an inverse?



No inv!

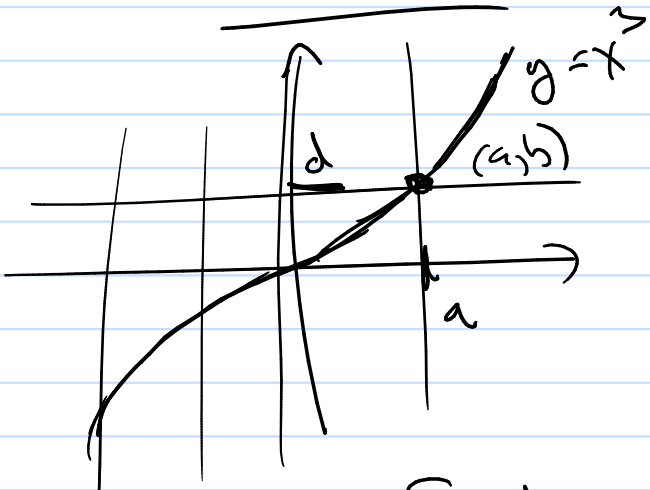


has an inv!



not one-to-one

f^{-1} does not exist



one-to-one functions

f^{-1} exists

if a graph passes the horiz. line test → f^{-1} exists.

Find it

① Swap all x's and y's

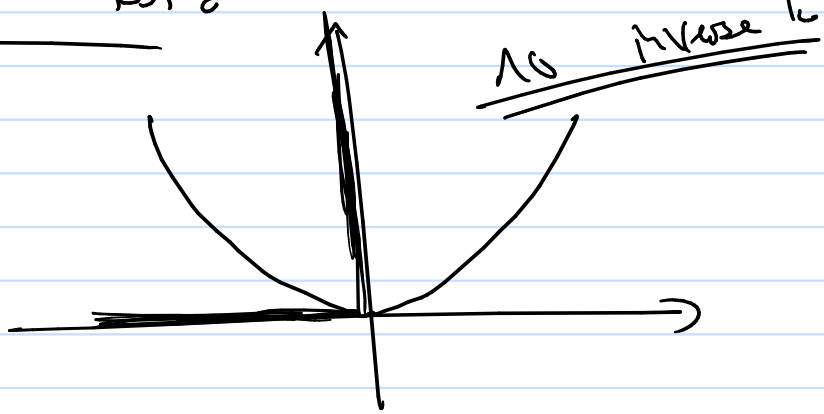
(ex) $y = x^3$ → find f^{-1}? $x = y^3$

inv. function!

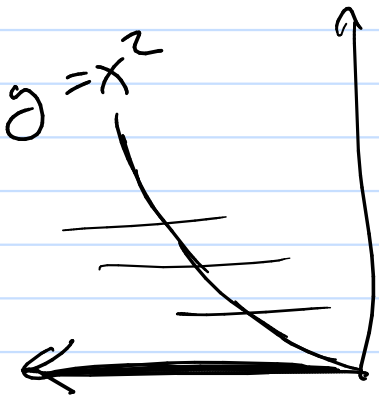
② the new $y = f^{-1}(x)$ solve for y (ex) $y = \sqrt[3]{x}$

Not passing horz. line test?

(ex) $y = x^2$



restrict domain and codomain to Quad #2



so this limited version does have f^{-1}

$$x = y^2$$

$$y^2 - x = 0$$

$$(y - \sqrt{x})(y + \sqrt{x}) = 0$$

$y = \sqrt{x}$ $\boxed{y = -\sqrt{x}} = f^{-1}(x)$