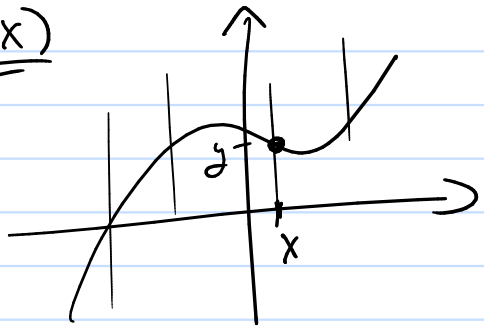
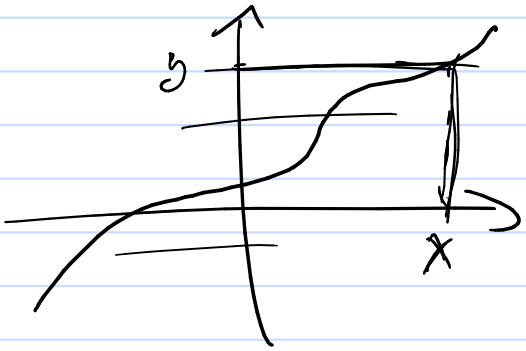
Math 112

$f(x)$



passes vertical test?
→ it is a function

$f(x)$ also passes horz. line test?



one to one function!
→ f^{-1} exists

Finding f^{-1}

$y = x^3 + x$

$y = x(x^2+1)$

passes horz. line test

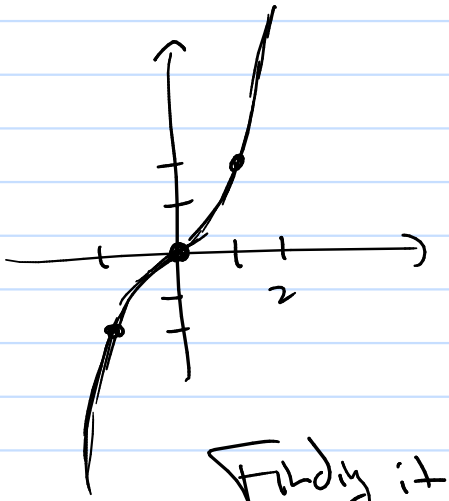
so $f^{-1}(x)$ exists

$f(-x) = (-x)^3 + (-x)$

$= -x^3 - x$

$= -f(x)$

sym about origin



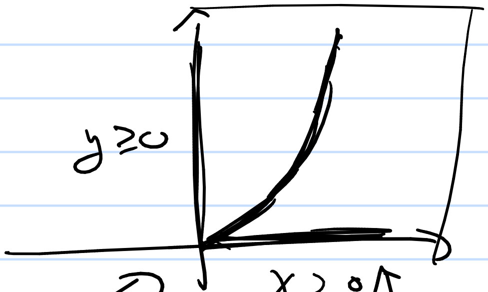
Finding it (swap y, x)

$x = y^3 + y$

do "math"

$y = f^{-1}(x)$ ←

(ex)



$y = x^2$

f^{-1} exists

find: $x = y^2$

$y^2 - x = 0$

$(y + \sqrt{x})(y - \sqrt{x}) = 0$

$y = -\sqrt{x}$ $\quad$ $\boxed{y = \sqrt{x}}$

so $\boxed{f(x) = x^2}$ $\quad$ $\boxed{f^{-1}(x) = \sqrt{x}}$

check:

$(f \circ f^{-1})(x) = f(f^{-1}(x))$

$= f(\sqrt{x}) = (\sqrt{x})^2 = x$ ✓

$(f^{-1} \circ f)(x) = f^{-1}(f(x))$

$= f^{-1}(x^2) = \sqrt{x^2} = |x| = x$ (b/c $x \geq 0$) ✓

Functions

1. polynomials → linear, quadratic, others: degree ≥ 3
2. rationals

ch 6

3. exponential
4. logarithms

Big Difference between exponential vs polynomial

polynomial: x^3 — x ← variable, 3 ← constant integer

$\text{base}^{\text{power}}$ (*) operation of exponentiation

exponential function: b^x — x ← variable, b ← positive real number constant

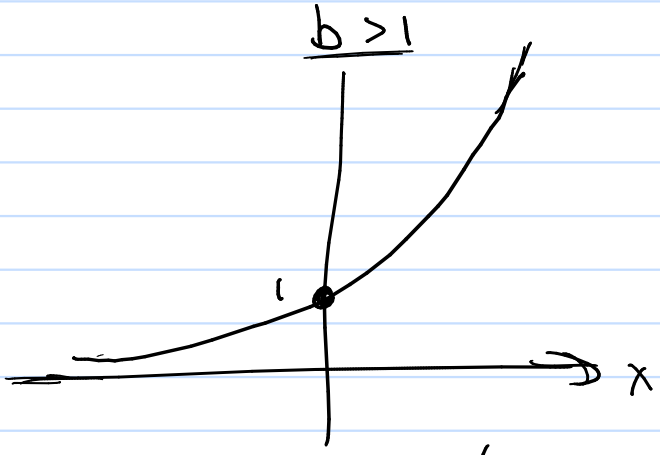
Example: Doubling

ex You win 1¢. Double it every day.

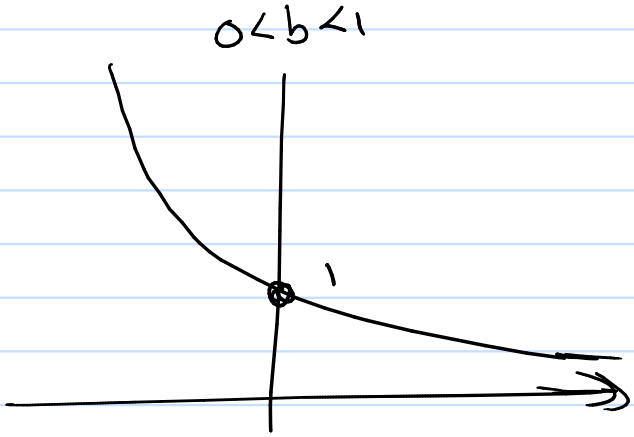
$t=1$	$a=1$
$t=2$	$a=2\cdot 1$
$t=3$	$a=2\cdot(2)=2^2$
$t=4$	$a=2(2^2)=2^3$
$t=5$	$a=2(2^3)=2^4$
	⋮
@ t	$a=2^{(t-1)}$

Exponential Functions: $f(x)=b^x$ $b>0$

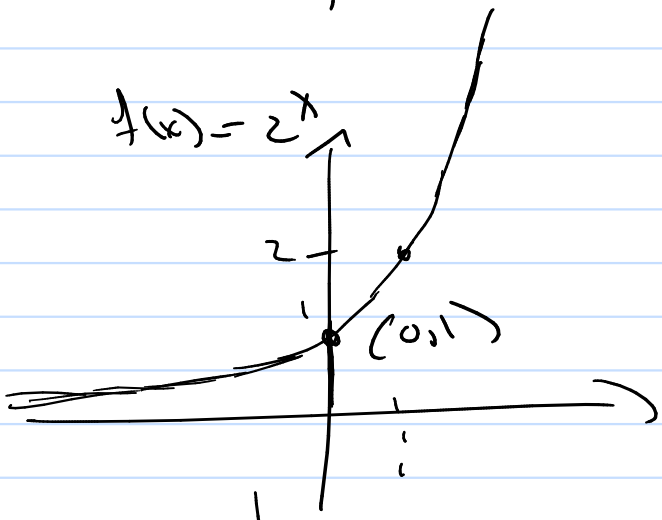
$b>1$



$0<b<1$

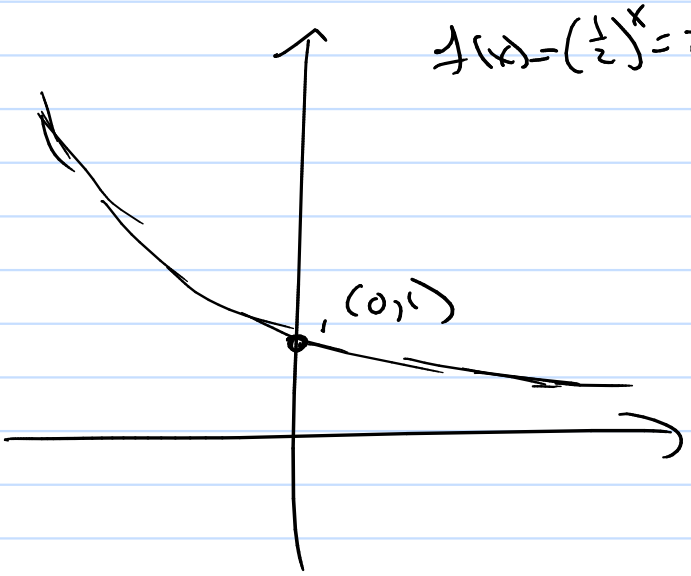


$f(x)=2^x$



x	2^x
-1	$\frac{1}{2}$
0	1
1	2

$f(x)=\left(\frac{1}{2}\right)^x=2^{-x}$



Given that $y = b^x$ passes Horz. line test ...

it will be invertible!

$f(x) = b^x$ $\quad$ $f^{-1}(x)$ exists

How to find it?

$y = b^x$ (swap y, x)

$x = b^y$

do "Math"

$y = f^{-1}(x)$ exists

$f(x) = b^x$	$f^{-1}(x) = \log_b(x)$
exponential function	logarithmic function

So $\log_b(x) = y$ means $x = b^y$

ex: $\log_2(8) = ?$ means $8 = 2^{(?)}$

$\log_2(8) = 3$ $\quad$ $8 = 2^3$

Inverse Properties: $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

① $\log_b(b^x) = x$

② $b^{\log_b x} = x$

$$\log_2\left(\frac{1}{16}\right) = \log_2\left(\frac{1}{2^4}\right) = \log_2\left(2^{-4}\right) = \boxed{-4}$$

$y = b^x$ non-integer powers?

$2^{(1/n)} = \sqrt[n]{2}$

$2^{a/b} = \left(2^{1/b}\right)^a = \left(\sqrt[b]{2}\right)^a = \left(2^a\right)^{1/b}$

$= \sqrt[b]{2^a}$