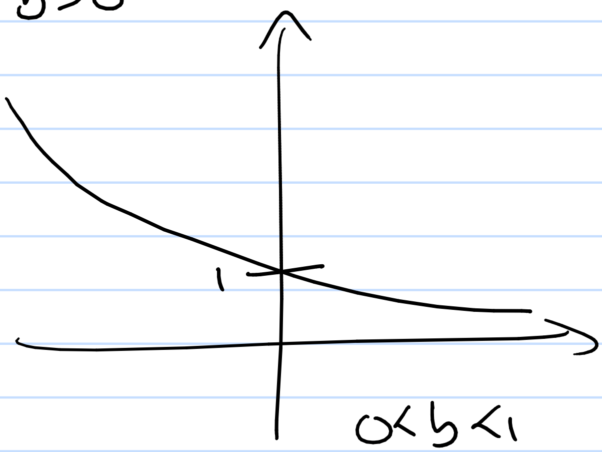
Math 112

Exponentials / logs

Exponential $y = b^x$ $\quad b>0$

$b>1$



$0<b<1$

Properties

① $b^a \cdot b^c = b^{a+c}$

② (i) $\dfrac{1}{b^{-a}} = b^a$ $\quad$ (ii) $b^{-c} = \dfrac{1}{b^c}$

ex $\dfrac{x^3 y}{x^{-4}} = x^4 x^3 y = x^7 y$

ex $\dfrac{y}{x^{1/2}} = y x^{-1/2}$

③ $\dfrac{b^a}{b^c} = b^{a-c}$

ex $\left(x^{-1/2} + x^{1/2}\right) = x^{-1/2}\left(1 + x^{\frac{1}{2}-\frac{-1}{2}}\right)$
$= x^{-1/2}(1+x)$

ex $\left(x^3 + x^2\right) = x^2\left(\dfrac{x^3}{x^2} + \dfrac{x^2}{x^2}\right)$
$= x^2\left(x^{3-2} + x^{2-2}\right)$
$= x^2(x + 1)$

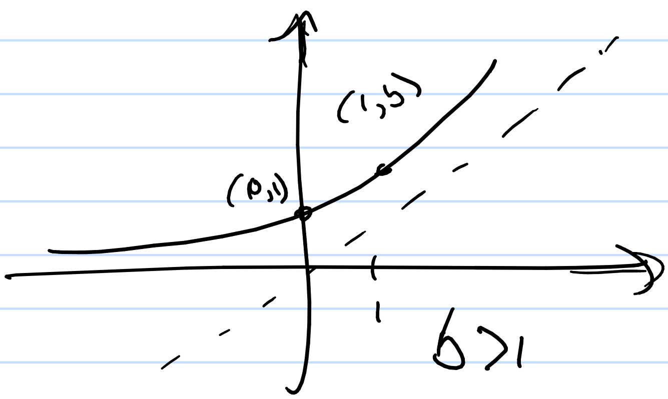
(4) $(b^a)^c = b^{a\cdot c}$

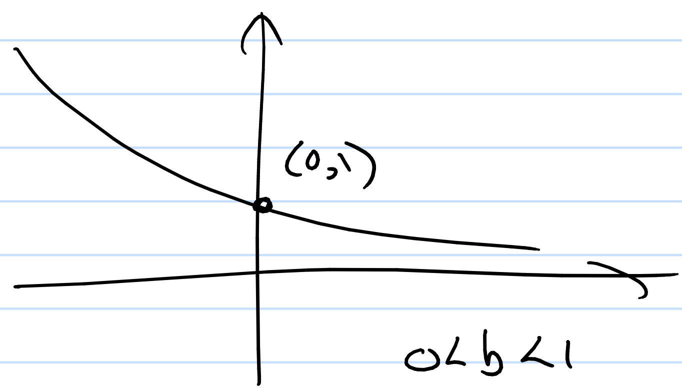
ex $(b^2)^3 = (b^2)(b^2)(b^2)$

$= (b\cdot b)(b\cdot b)(b\cdot b) = b^{2\cdot 3} = b^6$

ex $(x^{3/2})^4 = x^{\frac{3}{2}\cdot 4} = \boxed{x^6}$

Inverse of $f(x) = b^x$



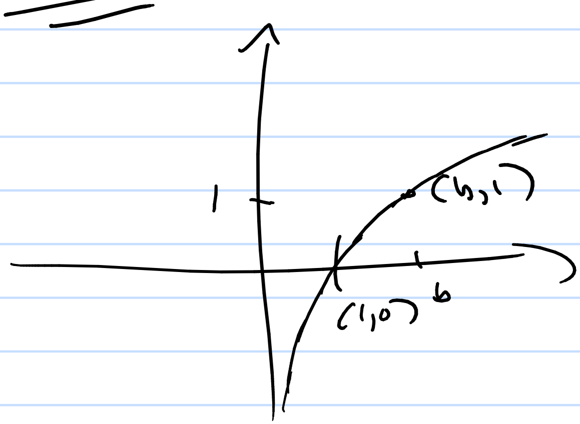


pass horiz. line test so.. f^{-1} exists!

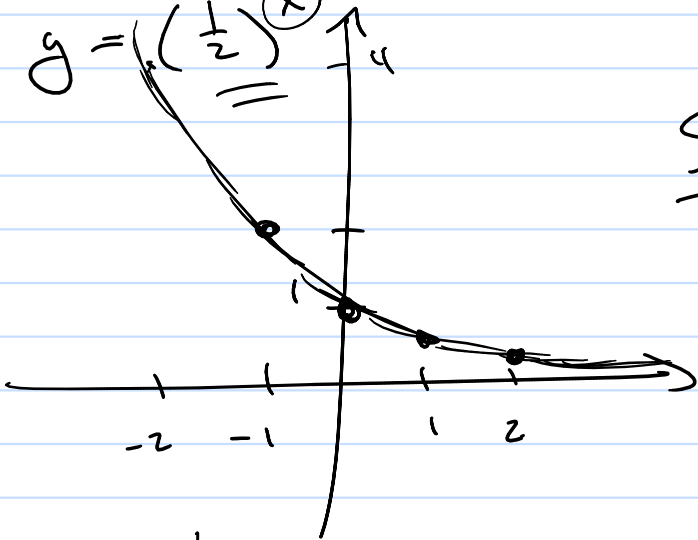
$y = b^x$

Find f^{-1}? (Swap x,y) $\rightarrow$ $x = b^y$

$y = \boxed{\quad}$



$y = \left(\frac{1}{2}\right)^{x}$

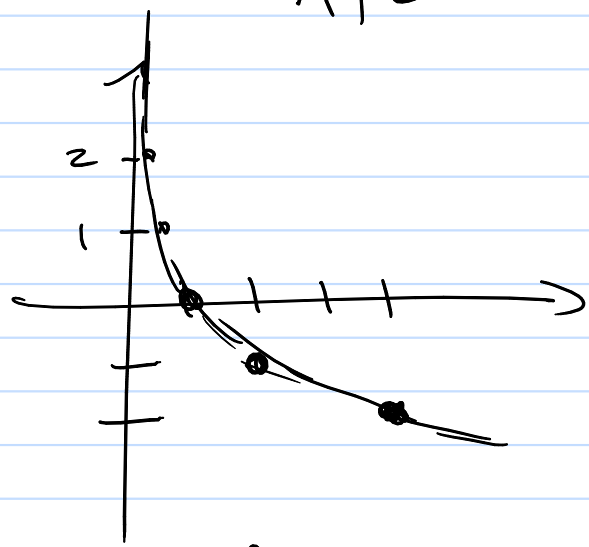


x	y
-2	4
-1	$= \left(\frac{1}{2}\right)^{-1} = 2$
0	1
1	$1/2$
2	$1/4$

Inverse?

Swap x, y

x	y
4	-2
2	-1
1	0
$1/2$	1
$1/4$	2



$y = \log_{\frac{1}{2}}(x)$

$\log_{1/2}(4) = -2$ b/c $\left(\frac{1}{2}\right)^{-2} = 4$

Def: $\log_b x = y$ b/c $b^y = x$

ex $\log_a 3 = \frac{1}{2}$ b/c $a^{1/2} = 3$

$\sqrt{a} = 3$

Note: $b = e$ (natural number)

$\log_e(x) = \ln(x)$ "natural log"

$b=10$ $\qquad \log_{10} x = \log(x)$

"common log"

Why?

$$\underbrace{231}_{\text{positional number in base 10}} = 2 \cdot 100 + 3 \cdot 10 + 1$$

$$= 2 \cdot 10^2 + 3 \cdot 10^1 + 1 \cdot 10^0$$

$\log(100) = \log_{10}(100) = 2$ $\qquad 10^2 = 100$

$\log(1000) = 3$

$\log(10{,}000) = 4$

Evaluation: $\ln(e^3) = 3$

$\log_e(2) = \boxed{\ln(2)} = ?$ $\qquad e^{?} = 2$

Properties of logs

① $\log_b(M \cdot n) = \log_b(M) + \log_b(n)$

② $\log_b\left(\frac{M}{n}\right) = \log_b(M) - \log_b(n)$

③ $\log_b(M^n) = n \log_b(M)$

$$\log\left(\frac{x^3y^2}{z^4}\right) = \log(x^3y^2) - \log(z^4)$$

$$= \log(x^3) + \log(y^2) - \log(z^4)$$

$$= 3\boxed{\log(x)} + 2\boxed{\log(y)} - 4\boxed{\log(z)}$$

$$\ln(3\cdot 4) = \ln(3) + \ln(4)$$