Math 112 $\quad f(x) = \sqrt{x^2+4} \qquad f(x) = \sqrt{x^2-4}$

$\boxed{\text{Q's}}$ **Domain of $(f \circ f)(x)$**

① $\sqrt{\left(\sqrt{x^2+4}\right)^2+4}$ vs ② $\sqrt{\left(\sqrt{x^2-4}\right)^2-4}$

$\boxed{\text{Domain}}$ $\quad \dfrac{1}{0} \leftarrow$ exclude! $\qquad \sqrt{\text{negative}} \leftarrow$ exclude!

① $\sqrt{\left(\sqrt{x^2+4}\right)^2+4}$

radical #1 $\quad x^2+4 \geq 0$

no zeros!

Domain: $+$, 0

radical #2 $\quad \left(\cancel{\sqrt{x^2+4}}\right)^{\cancel{2}}+4 \geq 0$

$x^2+4+4 \geq 0$

$x^2+8 \geq 0$

Domain: 0

radical #1 and radical #2 domains together $\rightarrow$ $\boxed{\text{all reals}}$

Fact: (a) $\sqrt{x^2} = |x|$

(b) $\left(\sqrt{x}\right)^2 = x, \; x \geq 0$

② $\sqrt{\left(\sqrt{x^2-4}\right)^2-4}$

radical #1 $\quad x^2-4 \geq 0$

$(x+2)(x-2) \geq 0$

sign table: $+ \quad - \quad +$; $-2 \quad 2$

radical #2 $\quad \left(\sqrt{x^2-4}\right)^2-4 \geq 0$

$x^2-4-4 \geq 0$

$\rightarrow x^2-8 \geq 0$

$(x+\sqrt{8})(x-\sqrt{8}) \geq 0$

$+ \quad - \quad +$; $-\sqrt{8} \quad \sqrt{8}$

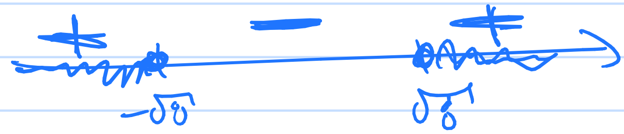
② $\sqrt{\left(\sqrt{x^2-4}\right)^2-4}$

radical #2: $\left(\sqrt{x^2-4}\right)^2-4 \geq 0$

$x^2-4-4 \geq 0$

$\rightarrow x^2-8 \geq 0$

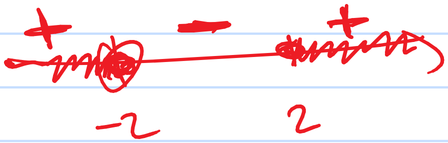
$(x+\sqrt{8})(x-\sqrt{8}) \geq 0$



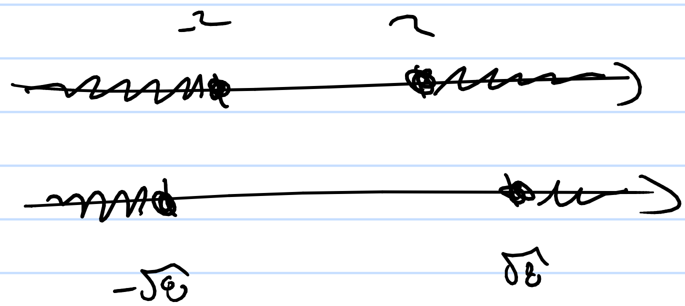
radical #1: $x^2-4 \geq 0$

$(x+2)(x-2) \geq 0$

sign table



together:



$$\boxed{(-\infty, -\sqrt{8}] \cup [\sqrt{8}, +\infty)}$$

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ex $\dfrac{\sqrt{x^2-4}}{x+3}$ Domain?

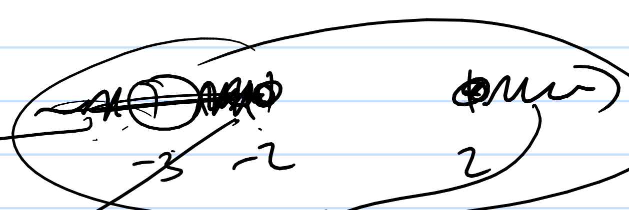
Denominator so $x+3=0$ denominator $= 0$ is bad

$x=-3$

its Domain is $x \neq -3$

radical $x^2-4 \geq 0$

$(x+2)(x-2) \geq 0$



together: $\boxed{(-\infty, -3) \cup (-3, 2] \cup [2, +\infty)}$

Exponentials / Logs

$f(x) = b^x$

$f(x) = \log_b x$

Properties:

$b^x b^y = b^{x+y}$ $\qquad$ $\log_b(xy) = \log_b x + \log_b y$

$\dfrac{b^x}{b^y} = b^{x-y}$ $\qquad$ $\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$(b^x)^y = b^{xy}$ $\qquad$ $\log_b(x^y) = y \log_b x$

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ex $3\log_2 x - 2\log_2(x+1) + \frac{1}{2}\log_2(x-1)$

$\log_2 x^3 - \log_2 (x+1)^2 + \log_2 (x-1)^{1/2}$

$= \log_2\left(\dfrac{x^3 (x-1)^{1/2}}{(x+1)^2}\right)$

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equations with exponential functions

use ① $\log_b x^y = y \log_b x$

② $q = r$

then $\log_b q = \log_b r$

ex $2^{x+1} = 3^{x-1}$

$\ln(2^{x+1}) = \ln(3^{x-1})$

$(x+1)\ln(2) = (x-1)\ln(3)$

$x\ln(2) + \ln(2) = x\ln(3) - \ln(3)$

$x\ln(2) - x\ln(3) = -\ln(2) - \ln(3)$

$x = \left(\dfrac{-\ln(2) - \ln(3)}{\ln(2) - \ln(3)}\right)$

Hint

$(x+1)7 = (x-1)5$

$7x + 7 = 5x - 5$

$\boxed{2x} = -12$

$\boxed{x = -6}$

Solving Equations w/ exponential functions

① Isolate Exponentials

② (a) ln( ) of both sides and

use $\ln(a^b) = b\ln(a)$

and solve

(b) $\boxed{\text{common base?}}$ equality of powers

$$\boxed{2^{x+1} = 2^{x^2-4}}$$

b/c $\log_b b^a = a$

$$\log_2 2^{x+1} = \log_2 2^{x^2-4}$$

$$\boxed{x+1 = x^2-4}$$

(1x)

$$12 + 6^{5x} = 15$$

$$6^{5x} = 3$$

$$\ln(6^{5x}) = \ln(3)$$

$$\frac{5x\cancel{\ln(6)}}{5\cancel{\ln(6)}} = \frac{\ln(3)}{\ln(6)} \cdot \frac{1}{5}$$

$$\boxed{x = \frac{\ln(3)}{5\ln(6)}}$$

ex $\quad 2^{x+5}\, 2^{x} = 64 \qquad$ b/c $\; 2^{a}\, 2^{b} = 2^{a+b}$

$$2^{(x+5)+x} = 64$$

$$2^{2x+5} = 64$$

$$2^{2x+5} = 2^{6}$$

So $\; 2x+5 = 6$

$$\boxed{x = \tfrac{1}{2}}$$

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$

ex $\quad 3^{2x}\, 2^{x} = 7$

$$\ln(3^{2x} \cdot 2^{x}) = \ln(7)$$

$$\underline{\ln(3^{2x})} + \underline{\ln(2^{x})} = \ln(7)$$

$$\underline{\underline{2x \ln(3)}} + \underline{\underline{x \ln(2)}} = \ln(7)$$

$$x\,(2\ln(3) + \ln(2)) = \ln(7)$$

$$x = \frac{\ln(7)}{2\ln(3) + \ln(2)} = \frac{\ln(7)}{\ln(18)}$$

b/c $\ln(a \cdot b) = \ln(a) + \ln(b)$
b/c $\ln(a^{b}) = b \ln(a)$

Special Cases $\quad$ quadratic type

$$a\,\square^{2} + b\,\square + c = 0$$

ex $\quad e^{2x} + e^{x} - 6 = 0$

(ex) $e^{2x}+e^{x}-6=0$

$(e^x)^2+(e^x)-6=0$ let $w=e^x$

$w^2+w-6=0$

$(w+3)(w-2)=0$

$w=-3$ $w=2$

$e^x=-3$ $e^x=2$

$(e^x+3)(e^x-2)=0$

$e^x=-3$ $e^x=2$

no soln $\ln(e^x)=\ln(2)$

$\boxed{x=\ln(2)}$

b/c $\log_b b^x = x$ so $\ln(e^x)=x$

$\log_e$