Math 112

$\boxed{\text{Q's}}$

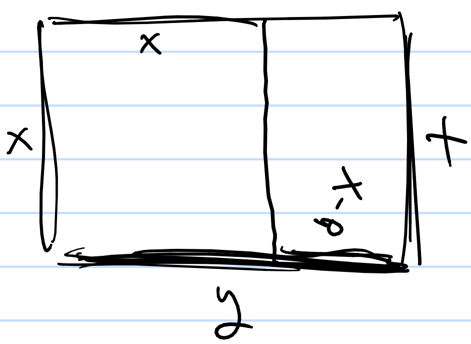
e? $\left(1+\frac{1}{n}\right)^n$ as $n\to\infty$ this goes to e.

$\boxed{e} = 2.718281828459045...$

It is called the natural number.

π

$C = \pi d \rightarrow \frac{C}{d} = \pi = 3.14...$



$\frac{y}{x} = \frac{x}{y-x}$ $\quad \phi$

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14 On the Richter Scale, the magnitude R of an earthquake of intensity I is

1 pt

$$R = \log_{10} \frac{I}{I_0}$$

$\boxed{\text{SF}}$ $8.25 = \log_{10}\left(\frac{I_{SF}}{I_0}\right)$

$\boxed{\text{Nephi}}$ $3.0 = \log_{10}\left(\frac{I_N}{I_0}\right)$

where I_0 is a reference intensity.

At 7:23am on March 19, 2003, an earthquake measuring 3.0 on the Richter scale occurred near the town of Nephi. An earthquake of that magnitude is often felt, but rarely causes damage.

By comparison, the earthquake that struck San Francisco in 1906 measured 8.25 on the Richter scale. It was ______ times as intensive as the Nephi earthquake.

Goal

$\boxed{\frac{I_{SF}}{I_N}} = \#$

$I_{SF} = \# I_N$

$\frac{B_H}{M_H} = \#$

$B_H = \# M_H$

Start!

$$8.25 = \log_{10}\left(\frac{I_{SF}}{I_0}\right) \rightarrow \boxed{10^{8.25} = \frac{I_{SF}}{I_0}}$$

$$3.0 = \log_{10}\left(\frac{I_N}{I_0}\right) \rightarrow 10^{3} = \frac{I_N}{I_0}$$

End!

$$\frac{I_{SF}}{I_N} = ?$$

$$\boxed{\log_b X = y \xleftrightarrow{\text{means}} b^y = X}$$

$$I_{SF} = I_0\, 10^{8.25} \qquad \text{so} \qquad \frac{I_{SF}}{I_N} = \frac{\cancel{I_0}\, 10^{8.25}}{\cancel{I_0}\, 10^{3}} = \boxed{10^{5.25}}$$

$$I_N = I_0\, 10^{3}$$

$\approx$ 180,000 times

13 The **pH** scale for acidity is defined by $\mathbf{pH} = -\log_{10}[\mathrm{H}^+]$ where $[\mathrm{H}^+]$ is the concentration of hydrogen ions measured in moles per liter (M). A substance has a hydrogen ion concentration of $[\mathrm{H}^+] = 8.6 \times 10^{-3}$M. Calculate the **pH** of the substance. 1 pt

$$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$$

$$\mathrm{pH} = \boxed{-\log_{10}(8.6 \times 10^{-3})}$$

$$\mathrm{pH} = -\left[\log_{10}(8.6) + \log_{10} 10^{-3}\right]$$

$$\log_b b^x = X$$

$$= -\left[\log_{10}(8.6) - 3\right]$$

$$\log_{10} 10^1 = 1$$

calculator?

$$-\log_{10}(\qquad)$$

$$2 \times 10^{3} = 2000.$$

$$3.1 \times 10^{-4} = 0.00031$$

change of base: $\log_a x = \frac{\log_b x}{\log_b a}$

ex $\log_{10}(0.0081) = \frac{\ln(0.0081)}{\ln(10)}$

Exponential Equations:

① Isolate exponential

② take $\ln(\ )$ of both sides

or $\log_b(\ )$

change of base:

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}} \qquad \boxed{a^x = b^{x \log_b(a)}}$$

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logarithmic Equations

① Isolate the logs (Need same bases!)

② rewrite $\log_b x = y \iff x = b^y$

and solve result.

ex $\log_6(x+4) + \log_6(3-x) = 1$

$\log_6[(x+4)(3-x)] = 1$

Domain

$3-x > 0$

$3 > x$

$x+4 > 0$

$x > -4$

Domain: $\log_b(x)$ must be positive!

$(x+4)(3-x) = 6^1$

$-x^2 - x + 12 = 6$

$-x^2 - x + 6 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3 \quad x = 2$ ← check if they are allowed.

Inequalities? (rational or exponential or logarithmic)

1. Make one side zero
2. Make other side to be one fraction (factor)
3. plot zeros of numerator, zeros of denom and make sign table

$\boxed{\text{Domain?}}$

Domain: $x+1>0 \to x>-1$

3. $x\log(x+1) \geq x$

① Make one side zero

$$x\log_{10}(x+1) - x \geq 0$$

② factor $(x)\left[\log_{10}(x+1) - 1\right] \geq 0$

③ Find zeros:

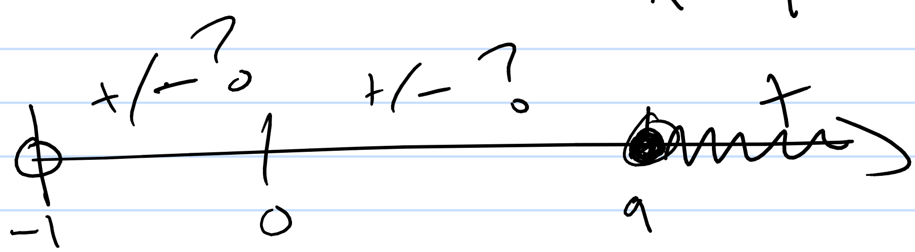
$x=0$ $\quad \log_{10}(x+1)-1=0$

$$\log_{10}(x+1) = 1$$

$$x+1 = 10^1$$

$$x = 9$$

④ Sign table



Finish!

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zero of denom

$$\frac{1}{\ln x + 1} \leq 1$$

Domain? $x>0$ and $\ln x+1=0$

b/c $\ln x$ needs only positives

$$\ln x = -1$$

$$\log_e x = -1$$

$$x = e^{-1}$$

$$x = \frac{1}{e}$$

so $x \neq \frac{1}{e}$

Domain

$\cancel{(\ln x+1)}\ \dfrac{1}{\cancel{(\ln(x)+1)}} \leq 1\,(\ln x+1)$

make one side zero

$$1 \leq \ln x + 1$$

$$0 \leq (\ln x)$$

Zeros? $\ln x = 0 \to x = 1$

Sign table