Math 112

Q's 6.3 #11

11 Find the inverse function of $f(x) = 9^{-6x+3} - 8$.

$f^{-1}(x) =$ [ ]

To find f^{-1} (swap x, y)

For $f(x)$: $y = 9^{-6x+3} - 8$

Swap! $x = 9^{-6y+3} - 8$

$x + 8 = 9^{-6y+3}$

$\log_9(x+8) = \log_9(9^{-6y+3})$

$\log_9(x+8) = -6y + 3$

$y = \boxed{\dfrac{\log_9(x+8) - 3}{-6}}$

$f^{-1} = \boxed{\dfrac{3 - \log_9(x+8)}{6}} = \boxed{\frac{1}{2} - \frac{1}{6}\log_9(x+8)}$

$= \boxed{\frac{1}{2} - \log_9 \sqrt[6]{x+8}}$

Exam 12 pobs @ 10pts

110pts = 100%

Ch 4 to ch 6

Ch 4 Rational Functions

4.1/4.2 Domains and Graphs of $r(x) = \frac{p(x)}{q(x)}$

2 pobs

(ex) $f(x) = \frac{3}{x} + \frac{x}{x-1}$ a) What is the domain?
b) Graph $f(x)$

$$f(x) = \frac{3}{x}\frac{(x-1)}{(x-1)} + \frac{x}{(x-1)}\cdot\frac{x}{x} = \frac{3x-3+x^2}{x(x-1)}$$

$$f(x) = \frac{x^2+3x-3}{x(x-1)}$$

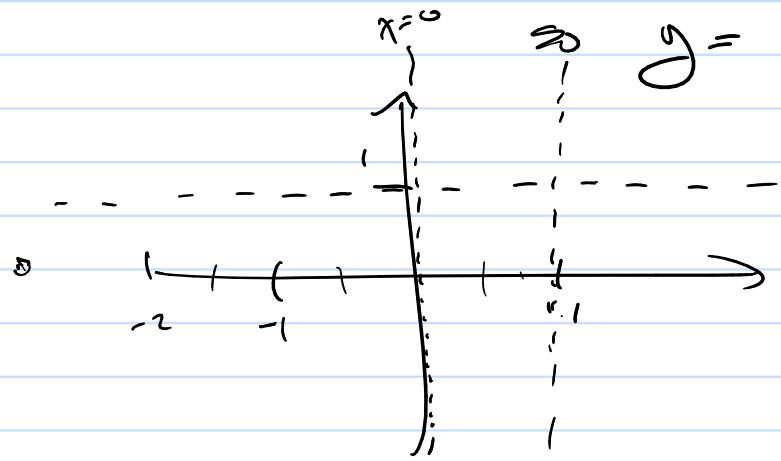
Domain: all reals except $x \neq 0$ and $x \neq 1$

Graph: ① Vertical asym or holes

$x=0$ and $x=1$ are vertical asym.

② horz. asym. $f(x) = \frac{x^2+3x-3}{x^2-x}$ deg = 2, deg = 2 same!

so $y = \frac{1}{1} = 1$ is the horz. asym.



③ Intercepts $x=0$ (nope!) (no y-axis int.)

$y=0 \rightarrow 0 = x^2+3x-3$

$x = \frac{-3}{2} \pm \frac{\sqrt{9+12}}{2}$

$x = -\frac{3}{2} \pm \sqrt{21}/2$

④ Table of values:

x	$y = \frac{x^2+3x-3}{x^2-x}$

Pick x's around the intercepts and vert. asymp

⑤ Finish the graph

4.3 Rational Equations and Inequalities (3 probs)

① ② } Solve the eqn

③ } Solve the inequality.

④ $\frac{1}{x} + \frac{1}{x^2-x} = 3$ Domain?: $x \neq 0, x \neq 1$

Factor: $\frac{1}{x} + \frac{1}{x(x-1)} = 3$ Common denom: $x(x-1)$

$$x(x-1)\left[\frac{1}{x} + \frac{1}{x(x-1)}\right] = [3]\,x(x-1)$$

$$(x-1) + 1 = 3x^2 - 3x \qquad x \neq 0, x \neq 1$$

$$3x^2 - 4x = 0$$

$$x(3x-4) = 0$$

$x = 0$ ~~$x = 0$~~ $x = 4/3$

(ex) $\frac{1}{x} + \frac{1}{x^2-x} \geq 3$

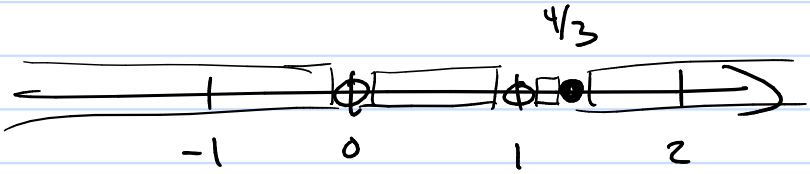
$\frac{1}{x} + \frac{1}{x(x-1)} - 3 \geq 0$

$\frac{(x-1) + 1 - 3x(x-1)}{x(x-1)} \geq 0$

$\frac{-3x^2 + 4x}{x(x-1)} \geq 0$

$\frac{x(-3x+4)}{x(x-1)} \geq 0$

Sign table



Ch 5 Compositia / Inverses

5.1 Compositia (2 probs) they will have several parts

(ex) $f(x) = x^2 + x$ $g(x) = \sqrt{x-1}$

a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-1})$

$= (\sqrt{x-1})^2 + \sqrt{x-1}$

$= x - 1 + \sqrt{x-1}$

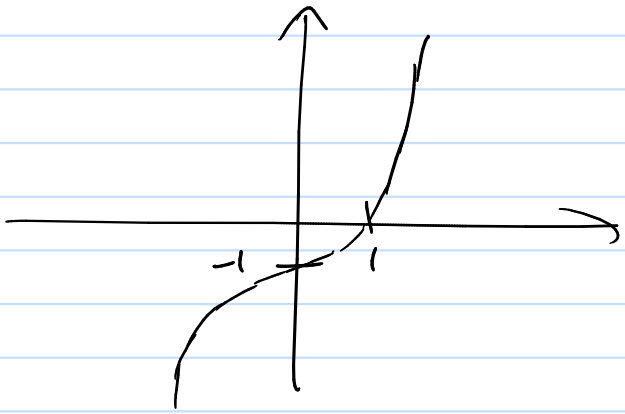
b) $(g \circ f)(1) = g(f(1)) = g(2) = \boxed{1}$

5.2 Inverses (1 prob)

① graph and does f^{-1} exist?
If so... find it!

(ex) $f(x) = x^3 - 1$

f^{-1} exists because graph passes horz. line test.



to find f^{-1} (swap x,y)

$y = x^3 - 1$

$x = y^3 - 1$

$x + 1 = y^3$

$y = \boxed{(x+1)^{1/3}} = \boxed{\sqrt[3]{x+1}}$

Ch 6 $f(x) = b^x$ and $f(x) = \log_b(x)$

6.1/6.2 Properties of b^x, $\log_b(x)$ & change of base

(1 prob) several parts

(ex) $\frac{2^{x+1}}{2^x} \cdot 4^{x-1} = \frac{2^{x+1}}{2^x} \cdot (2^2)^{x-1}$

$= \frac{2^{x+1}}{2^x} \cdot 2^{2x-2} = \frac{2^{3x-1}}{2^x} = \boxed{2^{2x-1}}$

(ex) $\log_2(x) + \log_2(x-1) = \log_2(x^2-1) + 3\log_2(x)$

$= \log_2 x(x-1) - \log_2(x^2-1) + \log_2 x^3$

$= \log_2 \frac{x(x-1)}{x^2-1} + \log_2 x^3$

$= \log_2\left[\frac{x^4(x-1)}{x^2-1}\right] = \log_2\left[\frac{x^4}{x+1}\right]$ $x \neq 1$

6.3/6.4 Eqns / Inequal. with b^x, $\log_b x$

3 probs

1. Equations w/ b^x
2. Equations w/ $\log_b x$
3. Inequality with $\log_b x$

(ex) $2^{x^2-3} - 3 = 2$

$2^{x^2-3} = 5$

$\log_2 2^{x^2-3} = \log_2(5)$

$x^2 - 3 = \log_2(5)$

$x^2 = 3 + \log_2(5)$

$x = \pm\sqrt{3+\log_2(5)}$

$x^2 - 3 = 1$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x^2 = 4$

$x = \pm\sqrt{4}$